BASIC CONCEPTS

1. **Matrix:** A matrix is a rectangular arrangement of numbers or functions arranged into a fixed number of rows and columns.

   A matrix is written inside brackets [ ]. Each entry in a matrix is called an element of the matrix.

   Order of Matrix: The dimension or order of matrix is defined by the number of rows and columns of that matrix. By conversion the dimension or order of a matrix is given by

   $$\text{No. of rows} \times \text{No. of columns}$$

   If a matrix have m rows and n columns then its order (dimension) is written as $m \times n$ and read as m by n.

2. **Row Matrix:** A matrix having one row and any number of column is called a row matrix. In other words, matrix of order $1 \times n$ is always a row matrix.

   e.g., $[a, b, c, d]_{1\times4}$ is row matrix.

3. **Column Matrix:** A matrix having any number of rows but only one column is called column matrix. In other words, a matrix of order $m \times 1$ is always a column matrix.

   e.g., $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4\times1}$ is column matrix.

4. **Square Matrix:** A matrix in which the number of rows is equal to the number of columns, say n, is called a square matrix of order n.

5. **Diagonal Elements:** The elements a_{ij} of a square matrix $A = [a_{ij}]_{n\times n}$ for which $i = j$, *i.e.*, the elements $a_{11}, a_{22}, \ldots, a_{nn}$ are called the diagonal elements and the line along which the diagonal elements lie, is called the principal diagonal or leading diagonal.

6. **Diagonal Matrix:** A square matrix $[a_{ij}]$ is said to be a diagonal matrix

   if $a_{ij} = 0$ for $i \neq j$.

   In other words, a square matrix is said to be a diagonal matrix, if its element not on principal diagonal are zero.

7. **Scalar Matrix:** A square matrix $A = [a_{ij}]_{n\times n}$ is called a scalar matrix, if

   (*i*) $a_{ij} = 0$ for all $i \neq j$ and

   (*ii*) $a_{ii} = c$ for all i, where $c \neq 0$.

   In other words, a square matrix is said to be scalar, if it is a diagonal matrix and enteries on its principal diagonal are equal.

8. **Identity Matrix:** A square matrix in which all non diagonal elements are zero and all diagonal elements are equal to 1 is called identity matrix.

i.e., $I = [a_{ij}]_{m \times n}$ is identity matrix if

$a_{ij} = 0 \quad \forall \, i \neq j$

and $a_{ij} = 1 \quad \forall \, i = j$

For example, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$ is identity matrix.

9. **Null or Zero Matrix:** A matrix whose all elements are zero is called a null matrix or a zero matrix *i.e.,* $A = [a_{ij}]_{m \times n}$ is null matrix if $a_{ij} = 0, \quad \forall \, i, j$

10. **Upper and Lower Triangular Matrices:** A square matrix $A = [a_{ij}]$ is called

(*i*) an upper triangular matrix, if $a_{ij} = 0$ for all $i > j$, *i.e.,* all eateries below principal diagonal are zero.

(*ii*) a lower triangular matrix, if $a_{ij} = 0$ for all $i < j$ *i.e.,* all eateries above principal diagonal are zero.

11. Two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ of the same order are equal, if

$a_{ij} = b_{ij}$ and all $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$

12. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices of the same order $m \times n$, then their sum $A + B$ is an $m \times n$ matrix such that

$(A + B)_{ij} = a_{ij} + b_{ij}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, 3, \ldots, n$

Following are the properties of matrix addition:

(*i*) Commutativity : If A and B are two matrices of the same order, then

$$A + B = B + A$$

(*ii*) Associativity : If A, B and C are three matrices of the same order, then

$$(A + B) + C = A + (B + C)$$

(*iii*) Existence of Identity : The null matrix is the identity element for matrix addition *i.e.,*

$$A + O = A + O = A$$

(*iv*) Existence of Inverse : For every matrix $A = [a_{ij}]_{m \times n}$ there exists a matrix $-A = [-a_{ij}]_{m \times n}$ such that

$$A + (-A) = O = (-A) + A$$

(*v*) Cancellation Laws : If A, B and C are three matrices of the same order, then

$$A + B = A + C \Rightarrow B = C \text{ and } B + A = C + A \Rightarrow B = C.$$

13. Let $A = [a_{ij}]$ be an $m \times n$ matrix and k be any number called a scalar. Then, the matrix obtained by multiplying every element of A by k is called the scalar multiple of A by k and is denoted by kA. Thus, $kA = [ka_{ij}]_{m \times n}$.

Following are the properties of scalar multiplication :

If A and B are two matrices of the same order and k, l are scalars, then

(*i*) $k(A + B) = kA + kB$ (*ii*) $(k + l)A = kA + lA$

(*iii*) $(kl)A = k(lA) = l(kA)$ (*iv*) $(-k)A = -(kA) = k(-A)$

(*v*) $1A = A$ (*vi*) $(-1)A = -A$

Note that a scalar matrix can be obtained by multiplying an identity matrix by a scalar.

14. If A and B are two matrices of the same order, then $A - B = A + (-B)$.

15. **Multiplication of Matrices :** Two matrices A and B are said to be defined for multiplication, if the number of columns of A (pre multiplier) is equal to the number of rows of B (post-multiplier).

For example, if the order of A (pre-multiplier) is $m \times n$ and the order of B (post-multiplier) is $n \times p$ then A and B is defined for multiplication and order of product of A and B denoted by AB is $m \times p$.

i.e.,

$$A_{m \times n} \times B_{n \times p} = AB_{m \times p}$$

Definition of Product : Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$ be two matrices then product of A and B denoted by AB is given as

$$AB = [c_{ij}]_{m \times p}$$

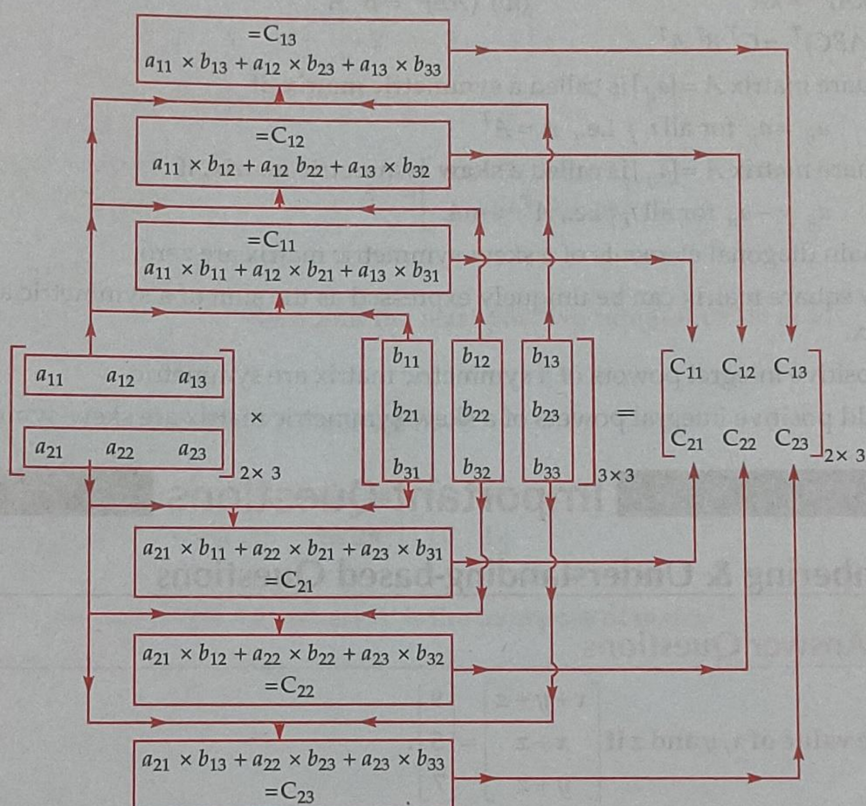
where $$C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj} = \sum_{r=1}^{n} a_{ir}b_{rj}$$

$$1 \le i \le m \text{ and } 1 \le j \le p.$$

Here, A is pre-multiplier or pre-factor.

B is post-multiplier or post-factor.

The diagram given below may help the students to understand the process of finding the product of two matrix:



Matrix multiplication has the following properties:

(*i*) Matrix multiplication is not commutative.

(*ii*) Matrix multiplication is associative *i.e.*, $(AB)\,C = A\,(BC)$ wherever both sides of the equality are defined.

(*iii*) Matrix multiplication is distributive over matrix addition *i.e.*, $A\,(B + C) = AB + AC$ and $(B + C)\,A = BA + CA$ wherever both sides of the equality are defined.

(*iv*) If A is an $m \times n$ matrix, then $I_m\, A = A = A\, I_n$

(*v*) If A is an $m \times n$ matrix and O is a null matrix, then

$$A_{m \times n} O_{n \times p} = O_{m \times p} \text{ and } O_{p \times m} \times A_{m \times n} = O_{p \times n}$$

i.e., the product of a matrix with a null matrix is a null matrix.

(*vi*) In matrix multiplication the product of two non-zero matrices may be a 'zero-matrix' *i.e.*, $AB = 0$, does not imply that at least one of the A or B should be zero.

16. If A is a square matrix, then we define $A^1 = A$ and $A^{n+1} = A^n . A$.

17. If A is a square matrix and $a_0, a_1, \ldots, a_n$ are constants, then

$$a_0A^n + a_1A^{n-1} + a_2A^{n-2} + \ldots + a_{n-1} A + a_n$$ is called a matrix polynomial.

18. **Transpose of a Matrix:** Let $A = [a_{ij}]$ be an $m \times n$ matrix. Then, the transpose of A, denoted by A^T, is an $n \times m$ matrix such that

$$(A^T)_{ij} = a_{ji} \text{ for all } i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.$$

i.e., the matrix obtained by interchanging rows into columns, of a given matrix A is called the transpose of A and is denoted by A^T or A'.

Following are the properties of transpose of a matrix:

(*i*) $(A^T)^T = A$ (*ii*) $(A + B)^T = A^T + B^T$

(*iii*) $(kA)^T = kA^T$ (*iv*) $(AB)^T = B^T A^T$

(*v*) $(ABC)^T = C^T B^T A^T$

19. A square matrix $A = [a_{ij}]$ is called a **symmetric matrix,** if

$$a_{ij} = a_{ji} \text{ for all } i, j \text{ i.e., } A = A^T$$

20. A square matrix $A = [a_{ij}]$ is called a **skew symmetric matrix,** if

$$a_{ij} = -a_{ji} \text{ for all } i, j \text{ i.e., } A^T = -A.$$

21. All main diagonal elements of a skew-symmetric matrix are zero.

22. Every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

23. All positive integral powers of a symmetric matrix are symmetric.

24. All odd positive integral powers of a skew-symmetric matrix are skew-symmetric.