CHAPTER 1 RATIONAL NUMBERS ** (EXPLANATION)

Students we have already studied about the rational numbers in class 7, now we are going to explore more about the rational numbers.

Kindly note down the following points in a register if you have or in any old copy or pages:

Rational numbers: A number is called rational if it can be expressed in the form p/q where p and q are integers (q ≠ 0).

Example: 2/1, 3/4, -5/6, 1/1, -9/10 etc. Since 0,-2,1,4 etc can be written in the form p/q, so they are also rational numbers.

Properties of rational numbers

1. **CLOSURE PROPERTY: a) Addition:** For any two rational numbers a and b, a + b is also a rational number. So we say that rational numbers are closed under addition.

Example: $\frac{3}{8} + (\frac{-5}{7}) = \frac{21+(-40)}{56} = \frac{-19}{56}$ (a rational number)

b) Subtraction: For any two rational numbers a and b, a - b is also a rational number. So we say that rational numbers are closed under subtraction.

Example: $\frac{-5}{7} - \frac{2}{3} = \frac{-29}{21}$ (a rational number)

c) Multiplication: For any two rational numbers a and b, a × b is also a rational number. So we say that rational numbers are closed under multiplication. example: $\frac{-2}{3} \times \frac{4}{5} = \frac{-8}{15}$ (a rational number)

d) Division: we note that $\frac{-5}{3} \div \frac{2}{5} = \frac{-25}{6}$ (a rational number)

but, $\frac{2}{5}$ **÷ 0 is not defined**. So we say that rational numbers are **not closed** under division.

2. **COMMUTATIVE PROPERTY: a) Addition:** For any two rational numbers a and b, a + b = b + a. so we say that addition is commutative for rational numbers.

   Example: $\frac{-2}{3} + \frac{5}{7} = \frac{1}{21}$ and $\frac{5}{7} + (\frac{-2}{3}) = \frac{1}{21}$

   so, $\frac{-2}{3} + \frac{5}{7} = \frac{5}{7} + (\frac{-2}{3})$

   b) Subtraction: We note that $\frac{2}{3} - \frac{5}{4} = \frac{-7}{12}$ but $\frac{5}{4} - \frac{2}{3} = \frac{7}{12}$

   so we say that subtraction is **not commutative** for rational numbers.

   c) Multiplication: For any two rational numbers a and b, a × b = b × a. so we say that multiplication is commutative for rational numbers.

Example: $\frac{-2}{3} \times \frac{5}{7} = \frac{-10}{21}$ and $\frac{5}{7} \times (\frac{-2}{3}) = \frac{-10}{21}$

So, $\frac{-2}{3} \times \frac{5}{7} = \frac{5}{7} \times (\frac{-2}{3})$

d) Division: We note that $\frac{-2}{3} \div \frac{5}{7} = \frac{-14}{15}$ and $\frac{5}{7} \div (\frac{-2}{3}) = \frac{15}{-14}$

we say that division is **not commutative** for rational numbers.

3. ASSOCIATIVE PROPERTY: a) Addition: For any three rational numbers a, b and c, a + (b + c) = (a + b) + c. So we say that addition is associative for rational numbers.

Example: $\frac{-2}{3} + [\frac{3}{5} + (\frac{-5}{6})] = \frac{-9}{10}$

$[\frac{-2}{3} + \frac{3}{5}] + (\frac{-5}{6}) = \frac{-9}{10}$

So, $\frac{-2}{3} + [\frac{3}{5} + (\frac{-5}{6})] = [\frac{-2}{3} + \frac{3}{5}] + (\frac{-5}{6})$

b) Subtraction: Subtraction is **not associative** for rational numbers.

Example: $\frac{-2}{3} - [\frac{-4}{5} - \frac{1}{2}] \neq [\frac{-2}{3} - \frac{-4}{5}] - \frac{1}{2}$

c) Multiplication: For any three rational numbers a, b and c,

a × (b × c) = (a × b) × c. So we say that multiplication is associative for rational numbers.

Example: $\frac{-7}{3} \times (\frac{5}{4} \times \frac{2}{9}) = \frac{-35}{54} = (\frac{-7}{3} \times \frac{5}{4}) \times \frac{2}{9}$

d) Division: Division is **not associative** for rational numbers.

Example: $\frac{1}{2} \div (\frac{-1}{3} \div \frac{2}{5}) \neq [\frac{1}{2} \div (\frac{-1}{3})] \div \frac{2}{5}$

4. DISTRIBUTIVITY OF MULTIPLICATION OVER ADDITION FOR RATIONAL NUMBERS: For rational numbers a, b and c,

a (b + c) = ab + ac

a (b – c) = ab - ac

Example: $\frac{-3}{4} \times \{\frac{2}{3} + (\frac{-5}{6})\} = (\frac{-3}{4} \times \frac{2}{3}) + (\frac{-3}{4} \times \frac{-5}{6}) = \frac{-1}{2} + \frac{5}{8} = \frac{1}{8}$

5. Additive Identity: Zero is called the additive identity of rational numbers. For any rational number a, a + 0 = 0 + a = a.

Example: $\frac{-3}{4} + 0 = 0 + (\frac{-3}{4}) = \frac{-3}{4}$

6. Multiplicative identity: One (1) is the multiplicative identity of rational numbers. For any rational number a, a × 1 = 1 × a = a.

Example: $\frac{-3}{4} \times 1 = 1 \times \left(\frac{-3}{4} \right) = \frac{-3}{4}$

7. Additive Inverse: For any rational number $\frac{a}{b}$, we have

$\frac{a}{b} + \left(-\frac{a}{b} \right) = \left(-\frac{a}{b} \right) + \frac{a}{b} = 0$. So we say that $\left(-\frac{a}{b} \right)$ is the additive inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is the additive inverse of $\left(-\frac{a}{b} \right)$.

Example: $\frac{2}{3} + \left(-\frac{2}{3} \right) = 0 = \left(-\frac{2}{3} \right) + \frac{2}{3}$

8. Multiplicative Inverse Or Reciprocal: Rational number $\frac{c}{d}$ is called the reciprocal or multiplicative inverse of rational number $\frac{d}{c}$.

So $\frac{d}{c} \times \frac{c}{d} = 1$

Example: $\frac{-5}{7} \times \frac{7}{-5} = 1$

Representation of rational numbers on the number line:

Students you have already learnt to represent rational numbers on the number line in class 7^{th}.

Rational numbers between two rational numbers: Between any two given rational numbers there are countless rational numbers.

Example:1. rational numbers between -2 and 0.

-2 can be written as $\frac{-20}{10}$ and 0 as $\frac{0}{10}$.

Thus we have $\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \ldots\ldots, \frac{-1}{10}$ between -2 and 0.

Example: 2. Ten rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.

We first convert $\frac{-5}{6}$ and $\frac{5}{8}$ to rational numbers with the same denominators.

$\frac{-5 \times 4}{6 \times 4} = \frac{-20}{24}$ and $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$

Thus we have $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \ldots., \frac{14}{24}$ between $\frac{-5}{6}$ and $\frac{5}{8}$.

*********HOME WORK************

1. Find using distributivity : a) $\{ \frac{7}{5} \times (\frac{-3}{12}) \} + \{ \frac{7}{5} \times \frac{5}{12} \}$

2. Write additive inverse of : a) $\frac{21}{121}$ b) $\frac{-7}{19}$

3. Write multiplicative inverse of : a) $\frac{5}{12}$ b) $\frac{-15}{17}$

4. Find using appropriate properties of rational numbers:

$$\frac{2}{5} \times \frac{-3}{7} - \frac{1}{14} - \frac{3}{7} \times \frac{3}{5}$$

5. Represent the following numbers on the number line:

a) $\frac{-5}{8}$ b) $\frac{8}{12}$

6. Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

Note: Students kindly do the homework in rough copy. We will also provide the exercises of this chapter next week that you all write in fair register if you have or in any old copy.