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UNITS AND DIMENSIONS

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## UNITS AND DIMENSIONS

## PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

## MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity.

## UNITS

All physical quantities are measured w.r.t. standard magnitude of the same physical quantity and these standards are called UNITS. eg. second, meter, kilogram, etc.

So the four basic properties of units are:-

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

## SET OF FUNDAMENTAL QUANTITIES

A set of physical quanties which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

## Physical Quantity

Mass
Length
Time
Temperature
Current
Luminous intensity
Amount of substance mol

## Units(CGS)

g
cm
s
${ }^{\circ} \mathrm{C}$
A
-
-

## Definition

The distance travelled by light in vacuum in $\frac{1}{299,792,458}$
second is called 1 metre.
The mass of a cylinder made of platinum-iridium alloy kept
at International Bureau of Weights and Measures is de-
fined as 1 kilogram.
The second is the duration of $9,192,631,770$ periods of
second is called 1 metre.
The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights and Measures is deThe second is the duration of $9,192,631,770$ periods of

Electric Current (A)

Thermodynamic Temperature (K)

Luminous Intensity (cd)
the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium133 atom.
If equal currents are maintained in the two parallel infinitely long wires of negligible cross-section, so that the force between them is $2 \times 10^{-7}$ newton per metre of the wires, the current in any of the wires is called 1 Ampere.

$$
1
$$

The fraction 273.16 of the thermodynamic temperature of triple point of water is called 1 Kelvin
1 candela is the luminous intensity of a blackbody of surface area $\frac{1}{600,000} \mathrm{~m}_{2}$ placed at the temperature of freezing platinum and at a pressure of 101,325 $\mathrm{N} / \mathrm{m}^{2}$, in the direction perpendicular to its surface. The mole is the amount of a substance that contains as many elementary entities as there are number of atoms in 0.012 kg of carbon- 12 .

There are two supplementary units too:

1. Plane angle (radian)

$$
\begin{aligned}
& \text { angle }=\operatorname{arc} / \text { radius } \\
& =\quad / \mathrm{r}
\end{aligned}
$$


2. SolidAngle (steradian)

## DERIVED PHYSICAL QUANTITIES

The physical quantities those can be expressed in terms of fundamental physical quantities are called derived physical quantities.eg. speed $=$ distance/time.

## DIMENSIONS AND DIMENSIONAL FORMULA

All the physical quantities of interest can be derived from the base quantities.

## DIMENSION

The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base.

To make it clear, consider the physical quantity "force".
Force $=$ mass $\times$ acceleration

$$
\begin{aligned}
& =\text { mass } \frac{\text { length } / \text { time }}{\text { time }} \\
& =\text { mass } \times \text { length } \times(\text { time })^{-2}
\end{aligned}
$$

So the dimensions of force are 1 in mass, 1 in length and -2 in time.
Thus [Force] $=$ MLT $^{-2}$

Similarly energy has dimensional formula given by

$$
[\text { Energy }]=\mathrm{ML}^{2} \mathrm{~T}^{-2}
$$

i.e. energy has dimensions, 1 in mass, 2 in length and -2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula.

## DIMENSIONAL EQUATION

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimen-sional equation.

## PRINCIPLE OF HOMOGENEITY

According to this principle, we can multiply physical quantities with same or different dimensional formu-lae at our convenience, however no such rule applies to addition and substraction, where only like physical quantites can only be added or substracted. e.g. If $\mathrm{P}+\mathrm{Q}$ $\mathrm{P} \& \mathrm{Q}$ both represent same physical quantity.

## Illustration :

Calculate the dimensional formula of energy from the equation $E=2 \mathrm{mv}^{2} . \quad \frac{1}{-}$
Sol. Dimensionally, $E=$ mass $\times(\text { velocity })^{2}$.

$$
1
$$

Since 2 is a number and has no dimension.
$o r,[E]=M \times \begin{aligned} & \mathrm{L}^{2} \\ & \mathrm{~T}^{2}=M L^{2} T^{-2} .\end{aligned}$

## Illustration :

Kinetic energy of a particle moving along elliptical trajectory is given by $K=s^{2}$ where $s$ is the distance travelled by the particle. Determine dimensions of .
Sol. $\quad K=s^{2}$
[]$=\frac{\left(\mathrm{M} \mathrm{L}^{2} \mathrm{~T}^{2}\right)}{\left(\mathrm{L}^{2}\right)}$
[]$=M^{L} L^{U} T^{-\iota}$
[ ] $=\left(M T^{-2}\right)$

## Illustration :

The position of a particle at time $t$, is given by the equation, $x(t)=\frac{\mathrm{v}_{0}}{\left(1-e^{-t}\right)}$, where $v_{0}$ is a constant and $>0$. The dimensions of $v_{0} \&$ are respectively.
(A) $M^{V} L^{1} T^{N} \& T^{-1}$
( $C^{*}$ ) $M^{U} L^{1} T^{-1} \& T^{-1}$

$=M L T \quad[]=M L \quad T$
(B) $M^{v} L^{1} T^{i}$ \& $T$
(D) $M^{1} L^{1} T^{-1} \& L T^{-L}$

Illustration :
The distance covered by a particle in time tis going by $x=a+b t+c t^{2}+d t^{3}$; find the dimensions of $a, b, c$ and $d$.

Sol. The equation contains five terms. All of them should have the same dimensions. Since $[x]=$ length, each of the remaining four must have the dimension of length.
Thus, $\quad[a]=$ length $=L$

| $[a]=$ length $=L$ |  |  |
| :--- | :--- | :--- |
| $[b t]=L$, | or | $[b]=L T^{-1}$ |
| $\left[c t^{2}\right]=L$, | or | $[c]=L T^{-2}$ |

and $\quad\left[d t^{3}\right]=L \quad$ or $\quad[d]=L T^{-3}$

## USES OF DIMENSIONAL ANALYSIS

## (I) TO CONVERT UNITS OFAPHYSICALQUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER :

It is based on the fact that,
Numerical value $\times$ unit $=$ constant
So on changing unit, numerical value will also gets changed. If $n_{1}$ and $n_{2}$ are the numerical values of a given physical quantity and $\mathrm{u}_{1}$ and $\mathrm{u}_{2}$ be the units respectively in two different systems of units, then

$$
\begin{aligned}
& \mathrm{n}_{1} \mathrm{u}_{1}=\mathrm{n}_{\mathrm{a}} \mathrm{u}_{2} \quad \mathrm{~b} \quad \mathrm{c} \\
& \text { n } \quad{ }_{2}{ }^{11} \frac{\mathrm{M}_{1}}{\mathrm{M}} \underset{2}{ } \quad \frac{\mathrm{~L}_{1}}{\mathrm{~L}_{2}} \quad \frac{\mathrm{~T}_{1}}{\mathrm{~T}}
\end{aligned}
$$

## Illustration

Young's modulus of steel is $19 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}$. Express it in dyne/cm ${ }^{2}$. Here dyne isthe CGS unit of force.
Sol. The unit of Young's modulus is $\mathrm{N} / \mathrm{m}^{2}$.
Force
This suggest that it has dimensions of $\overline{(\mathrm{dis} \tan \mathrm{ce})^{2}}$.
Thus, $\quad[Y]=\frac{[\mathrm{F}]}{\mathrm{L}^{2}}=\frac{\mathrm{MLT}^{2}}{\mathrm{~L}^{2}}=M L^{-1} T^{-2}$.
$\mathrm{N} / \mathrm{m}^{2}$ is in SI units,
So, $\quad 1 \mathrm{~N} / \mathrm{m}^{2}=(1 \mathrm{~kg})(1 \mathrm{~m})^{-1}(1 \mathrm{~s})^{-2}$
and $\quad 1$ dyne/ $\mathrm{cm}^{2}=(1 \mathrm{~g})(1 \mathrm{~cm})^{-1}(1 \mathrm{~s})^{-2}$
so, $\frac{1 \mathrm{~N} / \mathrm{m}^{2}}{1 \text { dyne } / \mathrm{cm}^{2}}=\frac{1 \mathrm{~kg}}{1 \mathrm{~g}} \frac{1 \mathrm{~m}}{1 \mathrm{~cm}} 1{\underset{\mathrm{~m}}{ }}_{1 \mathrm{~s}}^{1 \mathrm{~s}^{-2}} \quad=1000 \times \frac{1}{100} \quad \times \mathrm{l}=10$
or, $\quad 1 \mathrm{~N} / \mathrm{m}^{2}=10$ dyne $/ \mathrm{cm}^{2}$
or, $\quad 19 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}=19 \times 10^{11}$ dyne $/ \mathrm{m}^{2}$.

## Illustration :

The dimensional formula for viscosity of fluids is,

$$
=M^{1} L^{-1} T^{-1}
$$

Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $=M^{l} L^{-1} T^{-1}$
1 CGS units $=\mathrm{g} \mathrm{cm}^{-1} \mathrm{~s}^{-1}$
1 SI units $=\mathrm{kg} \mathrm{m}^{-1} \mathrm{~s}^{-1}$

$$
\begin{aligned}
& =1000 \mathrm{~g}(100 \mathrm{~cm})^{-1} \mathrm{~s}^{-1} \\
& =10 \mathrm{~g} \mathrm{~cm}^{-1} \mathrm{~s}^{-1}
\end{aligned}
$$

Thus, 1 Poiseuilli $=10$ poise

## Illustration :

A calorie is a unit of heat or energy and it equals about 4.2 J , where $1 J=1 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}^{2}$. Suppose we employ a system of units in which the unit of mass equals kg , the unit of length equals metre, the unit of time is second. Show that a calorie has a magnitude $4.2^{-1-2} 2$ in terms of the new units.
Sol. $\quad 1 \mathrm{cal}=4.2 \mathrm{~kg} \mathrm{~m}^{2} \mathrm{~s}^{-2}$

| SI | New system |
| :--- | :--- |
| $n_{l}=4.2$ | $n_{2}=?$ |
| $M_{l}=1 \mathrm{~kg}$ | $M_{2}=\mathrm{kg}$ |
| $L_{l}=1 \mathrm{~m}$ | $L_{2}=$ metre |
| $T_{1}=1 \mathrm{~s}$ | $T_{2}=$ second |

Dimensional formula of energy is $\left[M L^{2} T^{-2}\right]$
Comparing with $\left[M^{a} L^{b} T^{c}\right]$, we find that $a=1, b=2, c=-2$


## $\mathrm{kg} \mathrm{m} \quad \mathrm{s}$

(II) TO CHECK THE DIMENSIONALCORRECTNESS OFAGIVEN PHYSICALRELATION:

It is based on principle of homogeneity, which states that a given physical relation is dimensionally cor-rect if the dimensions of the various terms on either side of the relation are the same.
(i) Powers are dimensionless
(ii) $\sin , \mathrm{e}, \cos , \log$ gives dimensionless value and in above expression is dimensionless
(iii)We can add or subtract quantity having same dimensions.

## Illustration :

Let us check the dimensional correctness of the relation $v=u+a t$.
Here ' $u$ ' represents the initial velocity, 'v'represents the final velocity, 'a'the uniform acceleration and ' $t$ ' the time.
Dimensional formula of ' $u$ ' is $\left[M^{0} L T^{-1}\right]$
Dimensional formula of ' $v$ ' is $\left[M^{0} L T^{-1}\right]$
Dimensional formula of 'at' is $\left[M^{0} L T^{-2}\right][T]=\left[M^{0} L T^{-1}\right]$

Here dimensions of every term in the given physical relation are the same, hence the given physi-cal relation is dimensionally correct.

## Illustration :

Let us check the dimensional correctness of the relation

$$
x=u t+\frac{1}{2} a t^{2}
$$

Here ' $u$ ' represents the initial velocity, ' $a$ ' the uniform acceleration, ' $x$ ' the displacement and ' $t$ ' the time.
Sol.

$$
\begin{aligned}
& {[x]=L} \\
& {[u t]=\text { velocity } \times \text { time }=\frac{\text { length }}{} \times \text { time }=L} \\
& l_{a} t^{2}=[a t]=\text { accelecration } \times(\text { time })^{2}
\end{aligned}
$$

1
2 is a number hence dimentionless)

$$
=\frac{\text { velocity }}{\text { time }} \times(\text { time })^{2}=\frac{\text { length/time }}{\text { time }} \times(\text { time })^{2}=L
$$

Thus, the equation is correct as far as the dimensions are concerned.

## (III) TO ESTABLISHARELATION BETWEEN DIFFERENT PHYSICAL QUANTITIES :

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.

## Illustration :

Let us find an expression for the time period $t$ of a simple pendulum. The time period t may depend upon (i) mass $m$ of the bob of the pendulum, (ii) length of pendulum, (iii) acceleration due to gravity $g$ at the place where the pendulum is suspended.
Sol. Let (i) t
$m^{a}$
(ii) $t^{b}$
(iii) $\mathrm{t} \mathrm{g}^{\mathrm{c}}$

Combining all the three factors, we get
where $K$ is a dimensionless constant of proportionality.
Writing down the dimensions on either side of equation (i), we get

$$
[T]=\left[M^{a}\right]\left[L^{b}\right]\left[L T^{-2}\right]^{c}=\left[M^{a} L^{b+c} T^{-2 c}\right]
$$

Comparing dimensions, $a=0, b+c=0,-2 c=1$
$a=0, c=-1 / 2, b=1 / 2$


## Illustration :

When a solid sphere moves through a liquid, the liquid opposes the motion with a force $F$. The magnitude of $F$ depends on the coefficient of viscosity of the liquid, the radius $r$ of the sphere and the speed $v$ of the sphere. Assuming that $F$ is proportional to different powers of these quantities, guess a formula for $F$ using the method of dimensions.

Sol. Suppose the formula is $F=k^{a} r_{r}^{b} v^{c}$

$$
\begin{aligned}
& \text { Then, } M L T^{-2}=\left[\begin{array}{llllll}
M L & T & l^{-1} & L^{b} & \frac{\mathrm{~T}}{-} \\
\mathrm{T}
\end{array}\right. \\
& =M^{a} L^{-a+b+c} T^{-a-c}
\end{aligned}
$$

Equating the exponents of $M, L$ and $T$ from both sides,

$$
\begin{gathered}
\quad a=1 \\
-a+b+c=1 \\
-a-c=-2
\end{gathered}
$$

Solving these, $a=1, b=1$ and $c=1$
Thus, the formula for $F$ is $F=k r v$.

## Illustration :

If $P$ is the pressure of a gas and is its density, then find the dimension of velocity in terms of $P$ and.
(A) $P_{a}^{1 / 2-1 / 2}$
(B) $P^{1 / 2} 1 / 2$
(C) $P^{-1 / 2} 1 / 2$
(D) $P^{-1 / 2-1 / 2}$
[Sol. $v \begin{array}{llll} & v & P^{a} & b \\ & v=k P^{a} & b\end{array}$
$v=k P^{a b}$
$\left[L T^{-1}\right]=\left[M L^{-1} T^{-2}\right]^{a}\left[M L^{-3}\right]^{b}$ (Comparing dimensions)
$a=\frac{1}{2}, b=-\frac{1}{2}$
$[V]=\left[P^{1 / 2-1 / 2}\right]$
UNITS AND DIMENSIONS OF SOME PHYSICAL QUANTITIES

Quantity
Density
Force
Work
Energy
Power
Momentum
Gravitational constant
Angular velocity
Angular acceleration
Angular momentum
Moment of inertia
Torque
Angular frequency
Frequency
Period
Surface Tension
Coefficient of viscosity
Wavelength
Intensity of wave

SI Unit
$\mathrm{kg} / \mathrm{m}^{3}$
Newton (N)
Joule (J) (=N-m)
Joule(J)
Watt (W) (=J/s)
$\mathrm{kg}-\mathrm{m} / \mathrm{s}$
$\mathrm{N}-\mathrm{m}^{2} / \mathrm{kg}^{2}$
radian/s
radian/s ${ }^{2}$
$\mathrm{kg}-\mathrm{m}^{2} / \mathrm{s}$
$\mathrm{kg}-\mathrm{m}^{2}$
N -m
radian/s
Hertz (Hz)
s
N/m
$\mathrm{N}-\mathrm{s} / \mathrm{m}^{2}$
m
$\mathrm{W} / \mathrm{m}^{2}$

Dimensional Formula
$\mathrm{M} / \mathrm{L}^{3}$
$\mathrm{ML} / \mathrm{T}^{2}$
$\mathrm{ML}^{2} / \mathrm{T}^{2}$
$\mathrm{ML}^{2} / \mathrm{T}^{2}$
$\mathrm{ML}^{2} / \mathrm{T}^{3}$
ML/T
$\mathrm{L}^{3} / \mathrm{MT}^{2}$
$\mathrm{T}^{-1}$
T-2
$\mathrm{ML}^{2} / \mathrm{T}$
$\mathrm{ML}^{2}$
$\mathrm{ML}^{2} / \mathrm{T}^{2}$
T-1
T-1
T
$\mathrm{M} / \mathrm{T}^{2}$
M/LT
L
$\mathrm{M} / \mathrm{T}^{3}$

| Temperature | kelvin $(\mathrm{K})$ | K |
| :--- | :--- | :--- |
| Specific heat capacity | $\mathrm{J} /(\mathrm{kg}-\mathrm{K})$ | $\mathrm{L}^{2} / \mathrm{T}^{2} \mathrm{~K}$ |
| Stefan's constant | $\mathrm{W} /\left(\mathrm{m}^{2}-\mathrm{K}^{4}\right)$ | $\mathrm{M} / \mathrm{T}^{3} \mathrm{~K}^{4}$ |
| Heat | J | $\mathrm{ML}{ }^{2} / \mathrm{T}^{2}$ |
| Thermal conductivity | $\mathrm{W} /(\mathrm{m}-\mathrm{K})$ | $\mathrm{ML} / \mathrm{T}^{3} \mathrm{~K}$ |
| Current density | $\mathrm{A} / \mathrm{m}^{2}$ | $\mathrm{I} / \mathrm{L}^{2}$ |
| Electrical conductivity | $1 /-\mathrm{m}(=\mathrm{mho} / \mathrm{m})$ | $\mathrm{I}^{2} \mathrm{~T}^{3} / \mathrm{ML}^{3}$ |
| Electric dipole moment | $\mathrm{C}-\mathrm{m}$ | LIT |
| Electric field | $\mathrm{V} / \mathrm{m}(=\mathrm{N} / \mathrm{C})$ | $\mathrm{ML} / \mathrm{IT}^{3}$ |
| Potential (voltage) | volt $(\mathrm{V})(=\mathrm{J} / \mathrm{C})$ | $\mathrm{ML}^{2} / \mathrm{IT}^{3}$ |
| Electric flux | $\mathrm{V}-\mathrm{m}$ | $\mathrm{ML}^{3} / \mathrm{IT}^{3}$ |
| Capacitance | farad (F) | $\mathrm{I}^{2} \mathrm{~T}^{4} / \mathrm{ML}^{2}$ |
| Electromotive force | volt $(\mathrm{V})$ | $\mathrm{ML}^{2} / \mathrm{IT}^{3}$ |
| Resistance | ohm () | $\mathrm{ML}^{2} / \mathrm{I}^{2} \mathrm{~T}^{3}$ |
| Permittivity of space | $\mathrm{C}^{2} / \mathrm{N}-\mathrm{m}^{2}(=\mathrm{F} / \mathrm{m})$ | $\mathrm{I}^{2} \mathrm{~T}^{4} / \mathrm{ML}^{3}$ |
| Permeability of space | $\mathrm{N} / \mathrm{A}^{2}$ | $\mathrm{ML}^{2} \mathrm{I}^{2} \mathrm{~T}^{2}$ |
| Magnetic field | $\mathrm{Tesla}(\mathrm{T})\left(=\mathrm{Wb} / \mathrm{m}^{2}\right)$ | $\mathrm{M} / \mathrm{IT}^{2}$ |
| Magnetic flux | $\mathrm{Weber}(\mathrm{Wb})$ | $\mathrm{ML}^{2} / \mathrm{IT}^{2}$ |
| Magnetic dipole moment | $\mathrm{N}-\mathrm{m} / \mathrm{T}$ | $\mathrm{IL}^{2}$ |
| Inductance | Henry $(\mathrm{H})$ | $\mathrm{ML}^{2} / \mathrm{I}^{2} \mathrm{~T}^{2}$ |

## LIMITATIONS OF DIMENSIONAL ANALYSIS

(i) Dimension does not depend on the magnitude. Due to this reason the equation $x=u t+a t^{2}$ is also dimensionally correct. Thus, a dimensionally correct equation need not be actually correct.
(ii) The numerical constants having no dimensions connot be deduced by the method of dimensions.
(iii) This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.
SI Prefixes: The magnitudes of physical quantites vary over a wide range. The mass of an electron is $9.1 \times 10^{-31} \mathrm{~kg}$ and that of our earth is about $6 \times 10^{24} \mathrm{~kg}$. Standard prefixes for certain power of 10 . Table shows these prefixes :

| Power of 10 | Prefix | Symbol |
| :--- | :--- | :--- |
| 12 | tera | T |
| 9 | giga | G |
| 6 | mega | M |
| 3 | kilo | k |
| 2 | hecto | h |
| 1 | deka | da |
| -1 | deci | d |


| -2 | centi | c |
| :--- | :--- | :--- |
| -3 | milli | m |
| -6 | micro | $\mu$ |
| -9 | nano | n |
| -12 | pico | p |
| -15 | femto | f |

## ORDER-OF MAGNITUDE CALCULATIONS

If value of phycal quantity $P$ satisfy
$0.5 \times 10^{\mathrm{X}}<\mathrm{P} \quad 5 \times 10^{\mathrm{X}}$
$x$ is an integer
$x$ is called order of magnitude

## Illustration :

The diameter of the sun is expressed as $13.9 \times 10^{9}$ m. Find the order of magnitude of the diameter?
Sol. $\quad$ Diameter $=13.9 \times 10{ }^{9} \mathrm{~m}$
Diameter $=1.39 \times 10^{10} m$ order of magnitude is 10 .

## SYMBOLS AND THERE USUAL MEANINGS

The scientific group in Greece used following symbols.

|  | Theta |
| :---: | :---: |
| $\alpha$ | Alpha |
|  | Beta |
|  | Gamma |
|  | Delta |
|  | Delta |
|  | Mu |
|  | Lambda |
|  | Omega |
| , | Pi |
|  | Phi |
|  | epsilon |


|  | Psi |
| :---: | :---: |
|  | Roh |
|  | Nu |
|  | Eta |
|  | Sigma |
|  | Tau |
|  | Kappa |
|  | chi |
|  | Approximately <br> equal to |

## Solved Examples

Q. 1 Find the dimensional formulae of follwoing quantities:
(a) The surface tension $S$,
(b) The thermal conductivity k and
(c) The coefficient of vescosity

Some equation involving these quntities are
$\mathrm{S}=\frac{g r h}{2}$
$\mathrm{Q}=k \frac{A\left(\mathrm{z}_{2}-\right)_{1}}{d}$
$\operatorname{andF}=-\mathrm{A}_{x} \quad \frac{v_{2}-v_{1}}{-x} ;$
where the symbols have their usual meanings. ( - density, g - acceleration due to gravity, r - radius, h height, A - area, $1 \& 2$ - temperatures, t - time, d - thickness, $\mathrm{v}_{1} \& \mathrm{v}_{2}$ - velocities, $\mathrm{x}_{1} \& \mathrm{x}_{2}$ - positions.)

Sol.(a) $\mathrm{S}=\frac{g r h}{2}$
or $[\mathrm{S}]=[][\mathrm{g}] \mathrm{L}^{2}=\frac{M}{L} \cdot \frac{L}{T} \cdot \mathrm{~L}^{2}=\mathrm{MT}^{-2}$.
(b) $\mathrm{Q}=\mathrm{k} \frac{A\left(2-{ }_{1}\right) t}{d}$
or $\mathrm{k}=\frac{Q d}{A\left({ }_{2}-{ }_{1}\right) t}$.
Here, Q is the heat energy having dimension $\mathrm{ML}^{2} \mathrm{~T}^{-2}, 2^{-}{ }_{1}$ is temperature, A is area, d is thickness and t is time. Thus,
$[\mathrm{k}]=\frac{M L^{2} T^{-2}}{L^{2} K T}=\mathrm{MLT}^{-3} \mathrm{~K}^{-1}$.
(d) $\mathrm{F}=-\mathrm{h} \mathrm{A}^{\frac{v_{2}-v_{1}}{x^{2}-x_{1}}}$
$\operatorname{or~MLT}^{-2}=[] \mathrm{L} \frac{2 L / T}{L}=\left[\right.$ ] $\frac{L^{2}}{T}$
or, []$=\mathrm{ML}^{-1} \mathrm{~T}^{-1}$.
Q. 2 Suppose $A=B^{n} C^{m}$, where $A$ has dimensions LT, B has dimensions $L^{2} T^{-1}$, and C has dimensions $\mathrm{LT}^{2}$. Then the exponents n and m have the values:
(A) $2 / 3 ; 1 / 3$
(B) $2 ; 3$
(C) $4 / 5 ;-1 / 5$
(D*) $1 / 5 ; 3 / 5$
(E) $1 / 2 ; 1 / 2$

Sol. $\quad \mathrm{LT}=\left[\mathrm{L}^{2} \mathrm{~T}^{-1}\right]^{\mathrm{n}}\left[\mathrm{LT}^{2}\right]^{\mathrm{m}}$
$\mathrm{LT}=\mathrm{L}^{2 \mathrm{n}+\mathrm{m}} \mathrm{T}^{2 \mathrm{~m}-\mathrm{n}}$
$2 \mathrm{n}+\mathrm{m}=1$

## Solved Example

Q. 1 Given that A B 0 , but of three two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the vectors two having equal magnitude. Then the angles between vectors are given by -
(A) $30^{\circ}, 60^{\circ}, 90^{\circ}$
(B) $45^{\circ}, 45^{\circ}, 90^{\circ}$
(C) $45^{\circ}, 60^{\circ}, 90^{\circ}$
(D) $90^{\circ}, 135^{\circ}, 135^{\circ}$

Sol. (D) From polygon law, there vectors having summation zero, should from a closed polygon (triangle).
Since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. the triangle should be right angled triangle.


Angle between A and B is $90^{\circ}$
Angle between B and C is $135^{\circ}$
Angle between A and C is $135^{\circ}$
Q. 2 If a particle moves 5 m in + x-direction. Show the displacement of the particle-
(A) $5^{\mathrm{J}}$
(B) 5 i
(C) $-5^{\mathrm{J}}$
(D) 5 k

Sol.
Magnitude of vector $=5$
Unit vector in +x direction is i
displacement $=5 \mathrm{i}$
y


Hence correct answer is (B).
Q. 3 A car travels 6 km towards north at an angle of $45^{\circ}$ to the east then travels distnace of 4 km towards north at an angle of $135^{\circ}$ to the east. How far is its final position due east and due north? How far is the point from the strating point? What angle does the straight line joining its initial and final position makes with the east? What is the total distnace travelled by the car ?

Sol. Net movement along X - direction

$$
\begin{aligned}
& =(6-4) \cos 45^{\circ} \mathrm{i} \\
& =2 \times \frac{1}{\sqrt{2}}=\sqrt{2} \mathrm{~km}
\end{aligned}
$$



Net movement along Y - direction

$$
\begin{aligned}
& =(6+4) \sin 45^{\circ} \mathrm{j} \\
& =10 \times \sqrt{2 \overline{=}} 5 \sqrt{2} \mathrm{~km}
\end{aligned}
$$

Net movement form starting point (Total distance travelled)

$$
=6+4=10 \mathrm{~km}
$$

Angle which makes with the east direction

$$
\begin{aligned}
& \tan =\frac{Y-\text { component }}{X-\text { component }} \\
& =\frac{5 \sqrt{2}}{\sqrt{2}} \\
& =\tan ^{-1}(5)
\end{aligned}
$$

Q. 4 A body is moving with uniform speed $v$ on a horizontal circle in anticlockwise direction from Aas shown in figure. What is the change in velocity in (a) half revolution (b) first quarter revolution.


Sol. Change in velocity in half revolution
$\mathrm{v}=\mathrm{v}_{\mathrm{C}}-\mathrm{v}_{\mathrm{A}}$
$=v(-j)-v(j)$

## EXERCISE

## [SINGLE CORRECT CHOICE TYPE]

Q. 1 In the S.I. system, the unit of temperature is-
(A) degree centigrade
(B) Kelvin
(C) degree Celsius
(D) degree Fahrenheit
Q. 2 In the S.I. system the unit of energy is-
(A) erg
(B) calorie
(C) joule
(D) electron volt
Q. 3 The dimensions of the ratio of angular momentum to linear momentum is
(A) $\left[\mathrm{M}^{\mathrm{U}} \mathrm{LT}^{\mathrm{v}}\right]$
(B) $\left[\mathrm{MLT}^{-1}\right]$
(C) $\left[\mathrm{ML}^{\left.{ }^{2} \mathrm{~T}^{-1}\right]}\right.$
(D) $\left[\mathrm{M}^{-1} \mathrm{~L}^{-1} \mathrm{~T}^{-1}\right]$
Q. 4 If Force $=(x /$ density $)+\mathrm{C}$ is dimensionally correct, the dimension of x are -
(A) $\mathrm{MLT}^{-2}$
(B) $\mathrm{MLT}^{-3}$
(C) $\mathrm{ML}^{2} \mathrm{~T}^{-3}$
(D) $\mathrm{M}^{2} \mathrm{~L}^{-2} \mathrm{~T}^{-2}$
Q. 5 The dimensional formula for angular momentum is -
(A) $\mathrm{ML}^{2} \mathrm{~T}^{-2}$
(B) $\mathrm{ML}^{2} \mathrm{~T}^{-1}$
(C) $\mathrm{MLT}^{-1}$
(D) $\mathrm{M}^{0} \mathrm{~L}^{2} \mathrm{~T}^{-2}$
Q. 6 For $10^{(\mathrm{at}+3)}$, the dimension of a is-
(A) $\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{0}$
(B) $\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{1}$
(C) $\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{-1}$
(D) None of these
Q. $7 \quad$ The velocity of a moving particle depends upon time $t$ as $v=a t+\frac{b}{t}$. Then dimensional formula for $b$ is -
(A) $\left[\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{0}\right]$
(B) $\left[\mathrm{M}^{0} \mathrm{~L}^{1} \mathrm{~T}^{0}\right]$
(C) $\left[\mathrm{M}^{0} \mathrm{~L}^{1} \mathrm{~T}^{-1}\right]$
(D) $\left[\mathrm{M}^{0} \mathrm{~L}^{1} \mathrm{~T}^{-2}\right]$
Q. 8 The pairs having same dimensional formula -
(A)Angular momentum, torque
(B) Torque, work
(C) Plank's constant, boltzman's constant
(D) Gas constant, pressure
Q. 9 If $F=a x+b t 2+c$ where $F$ is force, $x$ is distance and $t$ is time. Then what is dimension of $b t \frac{a x c}{2}$ ?
(A) $\left[\mathrm{M} \mathrm{L}^{2} \mathrm{~T}^{-2}\right]$
(B) $\left[\mathrm{M} \mathrm{L} \mathrm{T}^{-2}\right]$
(C) $\left[\mathrm{M}^{0} \mathrm{~L}^{0} \mathrm{~T}^{0}\right]$
(D) $\left[\mathrm{M} \mathrm{L} \mathrm{T}^{-1}\right]$
Q. 10 If force, time and velocity are treated as fundamental quantities then dimensional formula of energy
will be
(A) [FTV]
(B) $\left[\mathrm{FT}^{2} \mathrm{~V}\right]$
(C) $\left[\mathrm{FTV}^{2}\right]$
(D) $\left[\mathrm{FT}^{2} \mathrm{~V}^{2}\right]$
Q. 11 Which of the following physical quantities do not have the same dimensions
(A) Pressure, Yongs modulus, stress
(B) Electromotive force, voltage, potential
(C) Heat, Work, Energy
(D) Electric dipole, electric field, flux

## EXERCISE-2 (Miscellaneous Exercise)

Q. 1 Taking force, length and time to be the fundamental quantities find the dimensions of
(I) Density
(II) Pressure
(III) Momentum and (IV) Energy
Q. 2 The frequency of vibration of a string depends on the length $L$ between the nodes, the tension $F$ in the string and its mass per unit length $m$. Guess the expression for its frequency from dimensional analysis.
Q. 3 The intensity of X-rays decreases exponentially according to the law $1 \mathrm{I}_{0} \mathrm{e} \times$, where $\mathrm{I}_{0}$ is the initial intensity of X-rays and $I$ is the intensity after it penetrates a distance $x$ through lead. If be the absorption coefficient, then find the dimensional formula for .
Q. 4 Find the dimensions of Planck's constant h from the equation $\mathrm{E}=\mathrm{h}$ where E is the energy and is the frequency.
Q. 5 If the velocity of light (c), gravitational constant (G) and the Planck's constant (h) are selected as the fundamental units, find the dimensional formulae for mass, length and time in this new system of units.
Q. 6 Find the dimensions of
(a) the specific heat capacity c ,
(b) the coefficient of linear expansion and
(c) the gas constant R.

Some of the equations involving these quantities are $\mathrm{Q}=\mathrm{mc}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right) l_{\mathrm{t}}=l_{0}\left[1+\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)\right]$ and $\mathrm{PV}=\mathrm{nRT}$.(Where $\mathrm{Q}=$ heat enegry, $\mathrm{m}=$ mass, $\mathrm{T}_{1} \& \mathrm{~T}_{2}=$ temperatures, $l_{\mathrm{t}}=$ length at temperature $\mathrm{t}{ }^{\circ} \mathrm{C}, l_{0}=$ length at temperature $0^{\circ} \mathrm{C}, \mathrm{P}=$ pressure, $\mathrm{v}=$ volume, $\mathrm{n}=$ mole )

