

**(Class X)**

**Exercise 1.1**

**Question 1:**

Use Euclid's division algorithm to find the HCF of:

**(i)** 135 and 225      **(ii)** 196 and 38220      **(iii)** 867 and 255

**Answer 1:**

**(i)** 135 and 225

Since  $225 > 135$ , we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Since remainder  $90 \neq 0$ , we apply the division lemma to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$$90 = 2 \times 45 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

**(ii)** 196 and 38220

Since  $38220 > 196$ , we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

**(iii)** 867 and 255

Since  $867 > 255$ , we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Since remainder  $102 \neq 0$ , we apply the division lemma to 255 and 102 to obtain

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$$255 = 102 \times 2 + 51$$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$$102 = 51 \times 2 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore,  
HCF of 867 and 255 is 51.

**Question 2:**

Show that any positive odd integer is of the form  $6q + 1$ , or  $6q + 3$ , or  $6q + 5$ , where  $q$  is some integer.

**Answer 2:**

Let  $a$  be any positive integer and  $b = 6$ .

Then, by Euclid's algorithm,  $a = 6q + r$  for some integer  $q \geq 0$ , and  $r = 0, 1, 2, 3, 4, 5$  because  $0 \leq r < 6$ .

Therefore,  $a = 6q$  or  $6q + 1$  or  $6q + 2$  or  $6q + 3$  or  $6q + 4$  or  $6q + 5$

Also,  $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$ , where  $k_1$  is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$ , where  $k_2$  is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$ , where  $k_3$  is an integer

Clearly,

$6q + 1, 6q + 3, 6q + 5$  are of the form  $2k + 1$ , where  $k$  is an integer.

Therefore,  $6q + 1, 6q + 3, 6q + 5$  are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form  $6q + 1$ , or  $6q + 3$ , or  $6q + 5$

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**Question 3:**

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

**Answer 3:**

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

**Question 4:**

Use Euclid's division lemma to show that the square of any positive integer is either of form  $3m$  or  $3m + 1$  for some integer  $m$ .

[**Hint:** Let  $x$  be any positive integer then it is of the form  $3q$ ,  $3q + 1$  or  $3q + 2$ . Now square each of these and show that they can be rewritten in the form  $3m$  or  $3m + 1$ .]

**Answer 4:**

Let  $a$  be any positive integer and  $b = 3$ .

Then  $a = 3q + r$  for some integer  $q \geq 0$

And  $r = 0, 1, 2$  because  $0 \leq r < 3$

Therefore,  $a = 3q$  or  $3q + 1$  or  $3q + 2$  Or,

$$\begin{aligned} a^2 &= (3q)^2 \text{ or } (3q + 1)^2 \text{ or } (3q + 2)^2 \\ &= (3q)^2 \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4 \\ &= 3 \times (3q^2) \text{ or } 3 \times (3q^2 + 2q) + 1 \text{ or } 3 \times (3q^2 + 4q + 1) + 1 \\ &= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1 \end{aligned}$$

Where  $k_1$ ,  $k_2$ , and  $k_3$  are some positive integers

Hence, it can be said that the square of any positive integer is either of the form  $3m$  or  $3m + 1$ .

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**Question 5:**

Use Euclid's division lemma to show that the cube of any positive integer is of the form  $9m$ ,  $9m + 1$  or  $9m + 8$ .

**Answer 5:**

Let  $a$  be any positive integer and  $b = 3$

$a = 3q + r$ , where  $q \geq 0$  and  $0 \leq r < 3$

$a = 3q$  or  $3q + 1$  or  $3q + 2$

Therefore, every number can be represented as these three forms.

There are three cases.

**Case 1:** When  $a = 3q$ ,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Where  $m$  is an integer such that  $m = 3q^3$

**Case 2:** When  $a = 3q + 1$ ,

$$a^3 = (3q + 1)^3$$

$$a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$a^3 = 9(3q^3 + 3q^2 + q) + 1$$

$$a^3 = 9m + 1$$

Where  $m$  is an integer such that  $m = (3q^3 + 3q^2 + q)$

**Case 3:** When  $a = 3q + 2$ ,

$$a^3 = (3q + 2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$a^3 = 9m + 8$$

Where  $m$  is an integer such that  $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form  $9m$ ,  $9m + 1$ , or  $9m + 8$ .