POLYNOMIALS

POLYNOMIALS

Objective: To study

- Polynomials of degree 1, 2 &3
- The zeroes of polynomial p(x)
- Relationship between zeroes and coefficients of a polynomial
- The division algorithm of a polynomial

Do we remember this???

POLYNOMIAL

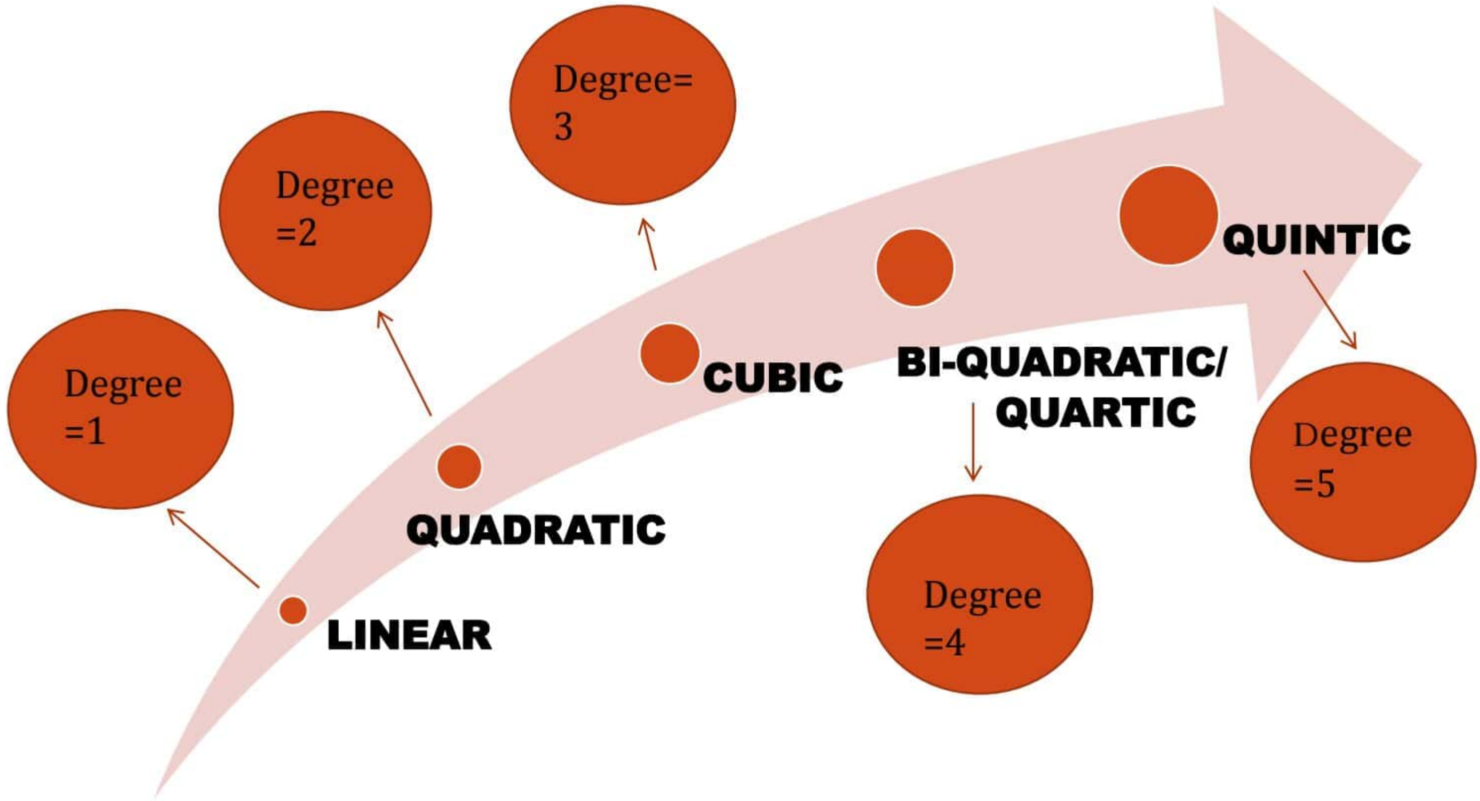
VARIABLES
(one, two, three, many)

DEGREE
(1, 2, 3, 4, 5)

VALUE OF A POLYNOMIAL

ZERO OF A POLYNOMIAL

DEGREE OF A POLYNOMIAL



VALUE OF A POLYNOMIAL

If $p(x)$ is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in $p(x)$ is called the **value of p(x) at x=k.**

Example: $p(x) = x^2 + 2$

Then, $p(-4) = (-4)^2 + 2$

$= 16 + 2$

$= 18$

ZERO OF A POLYNOMIAL

- A real number k, is said to be a zero of the polynomial p(x), if p(k)=0
- **Example:** For a linear polynomial p(x)= 2x+3

  Then , p(k)= 0 gives us

  2k+3 = 0. i.e. k= − 3/2
- In general, if k is a zero of p(x)= ax+b, then,

  k= − b/a

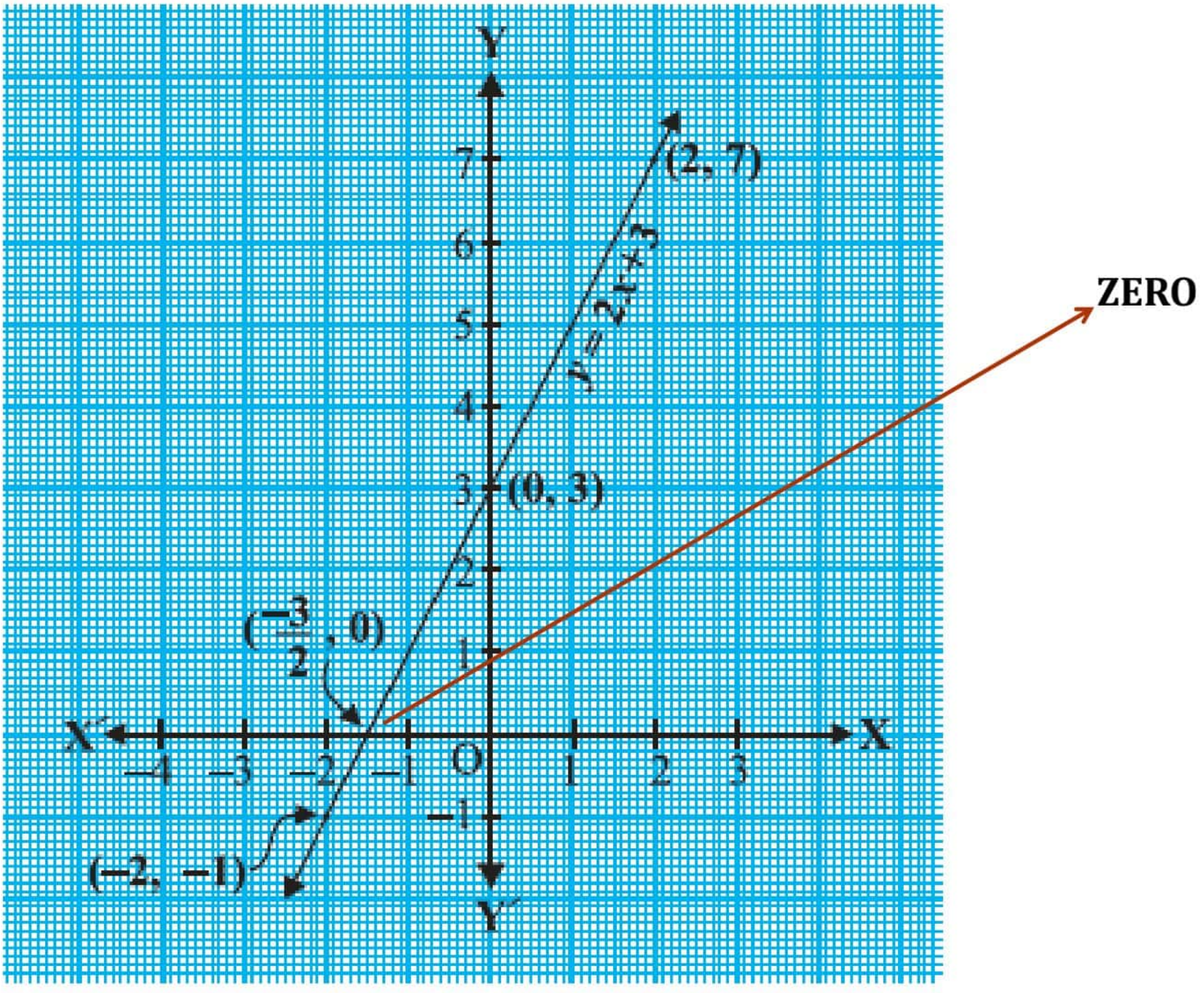
WHAT'S NEW THIS TIME?

- Geometrical meaning of the zeroes of a polynomial
- Relationship between the zeroes and coefficients of a polynomial
- Division Algorithm for Polynomials

Geometrical meaning of the zeroes of a polynomial

LINEAR POLYNOMIAL

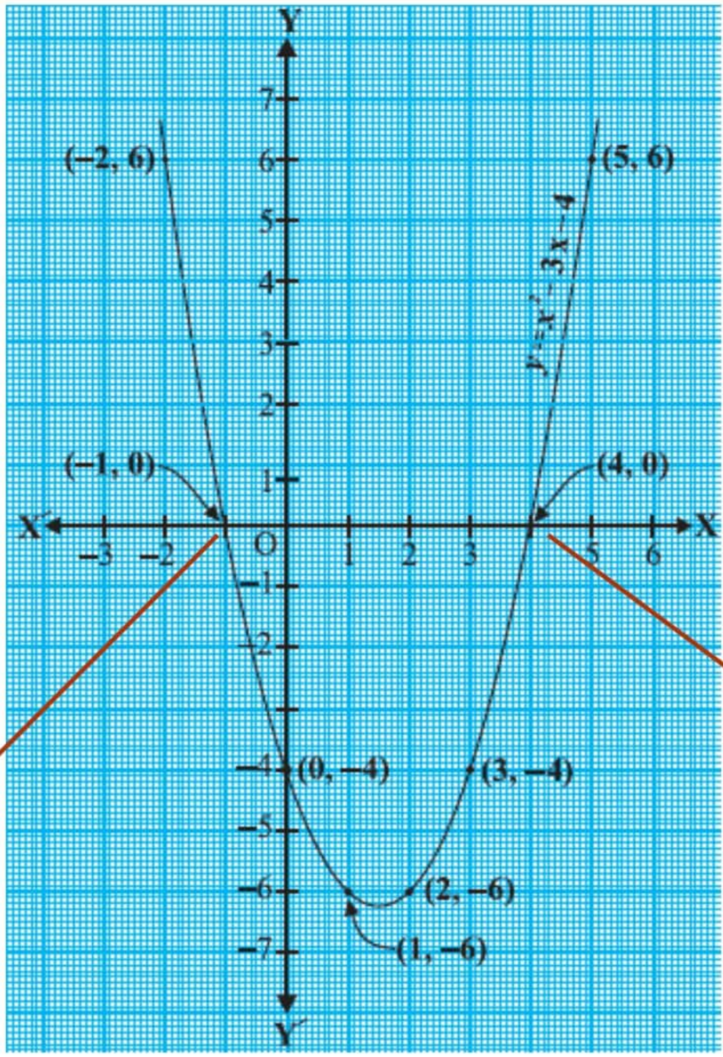
Plot the graph of the polynomial $y = 2x+3$


Y
(2, 7)
y = 2x+3
(0, 3)
$(\frac{-3}{2}, 0)$
ZERO
X'
X
O
(−2, −1)
Y'

Geometrical meaning of the zeroes of a polynomial

QUADRATIC POLYNOMIAL

Plot the graph of the polynomial $y = x^2 - 3x + 4$



FIRST ZERO

SECOND ZERO

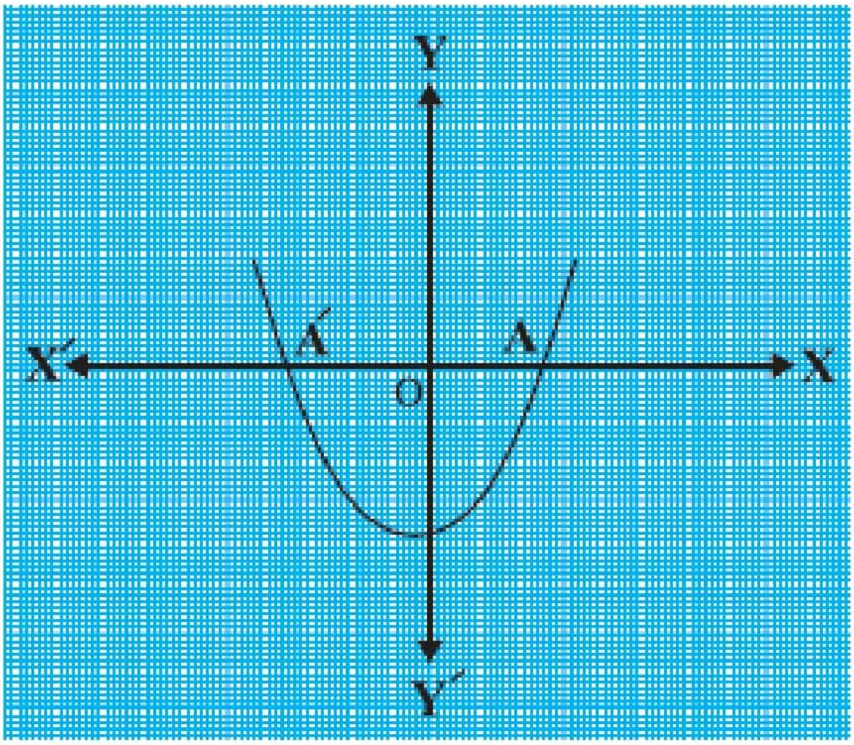
Special cases in quadratic polynomials

Question:

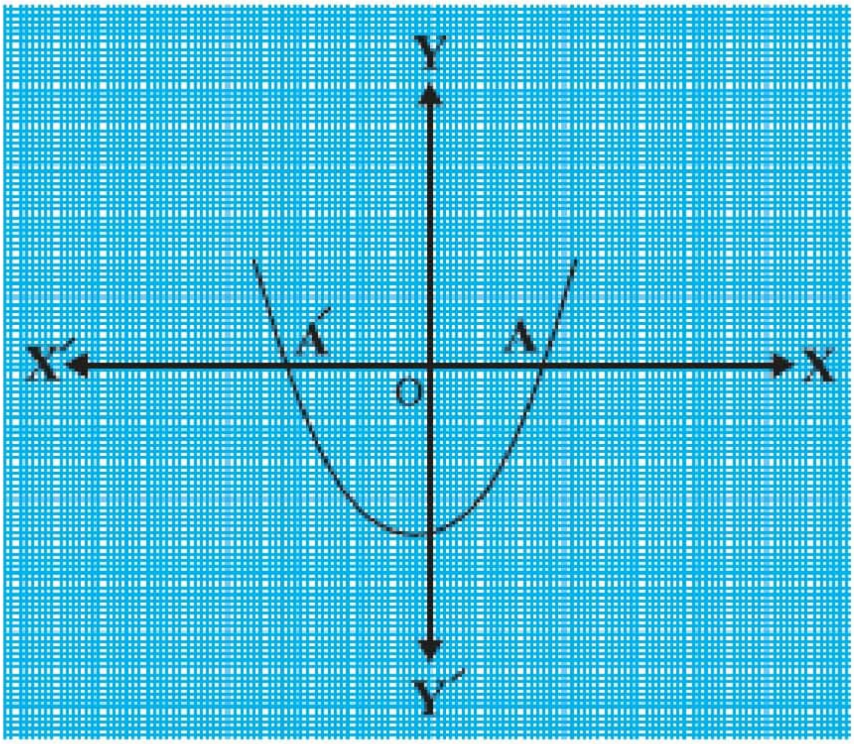
How many zeroes do the following polynomials have?

How many zeroes do the following polynomials have ?

(a)



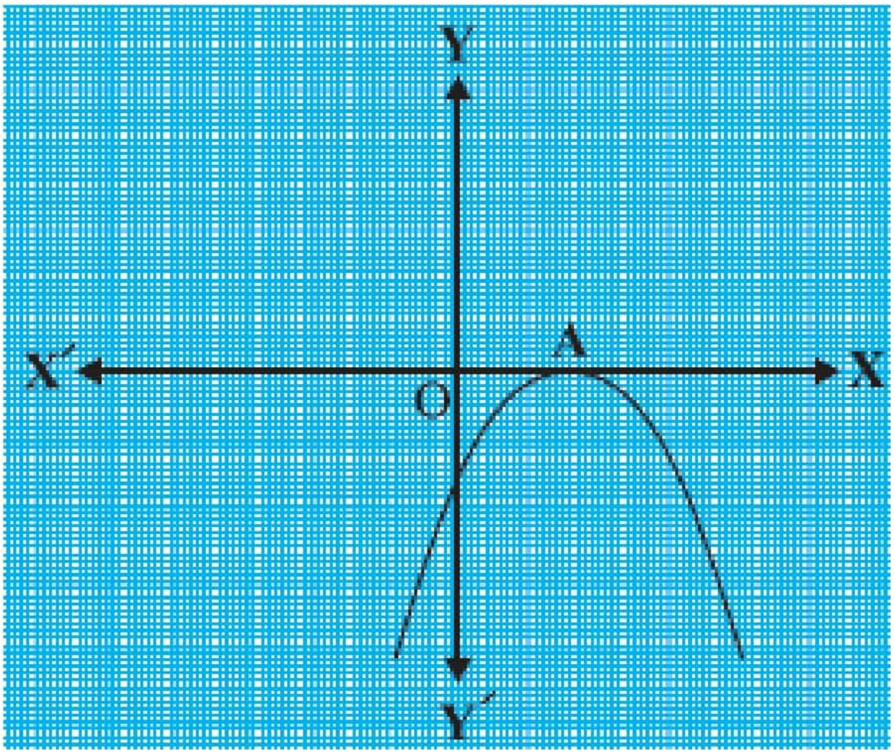
(a)



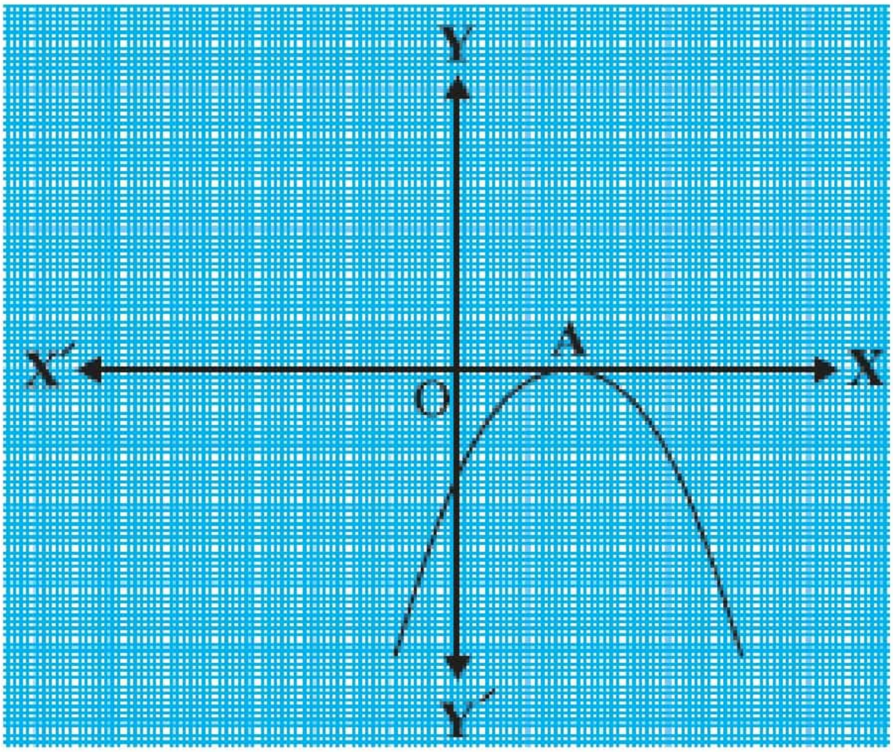
Answer: 2

How many zeroes do the following polynomials have ?

(b)



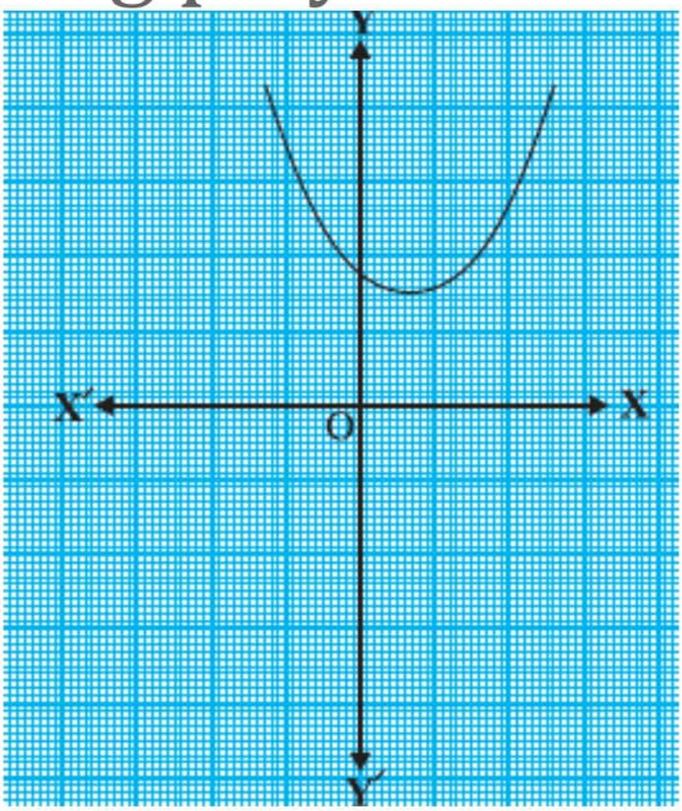
(b)



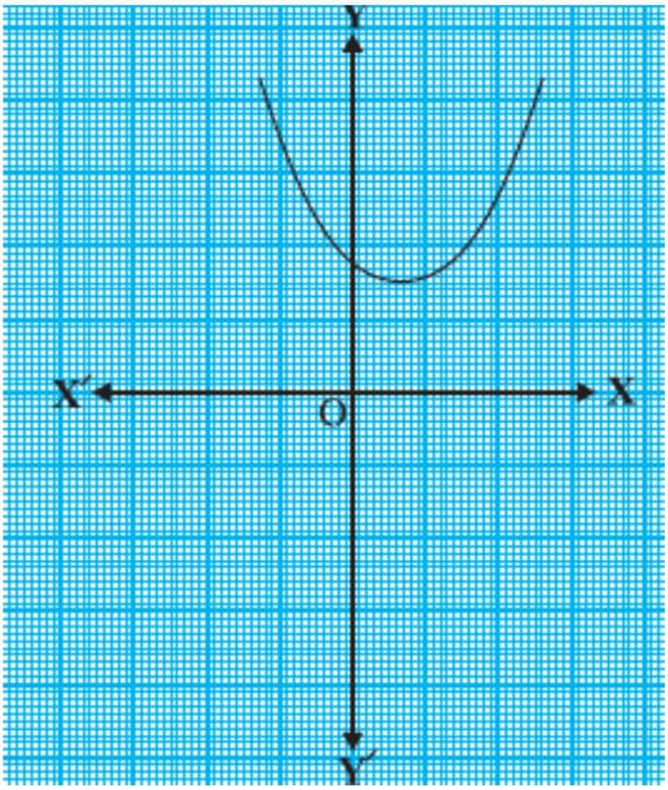
Answer: 1

How many zeroes do the following polynomials have ?

(c)



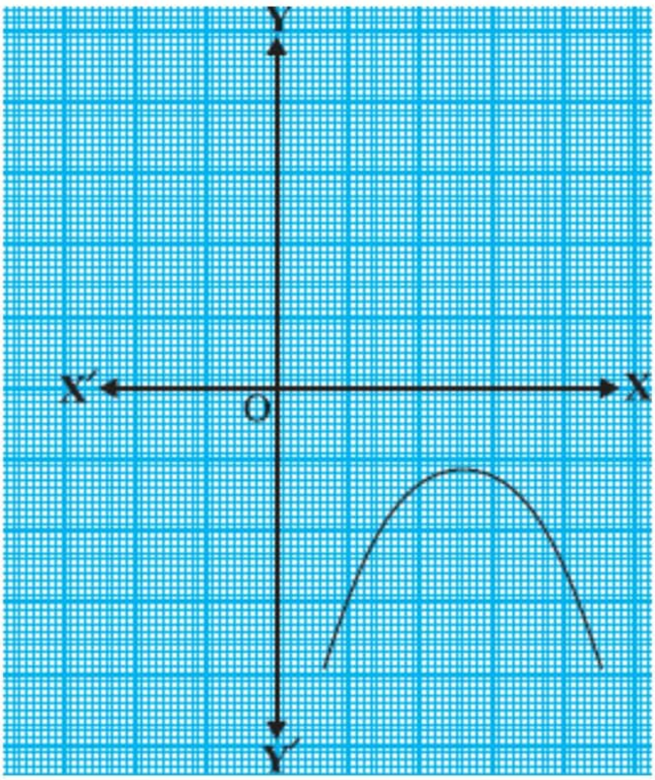
(c)



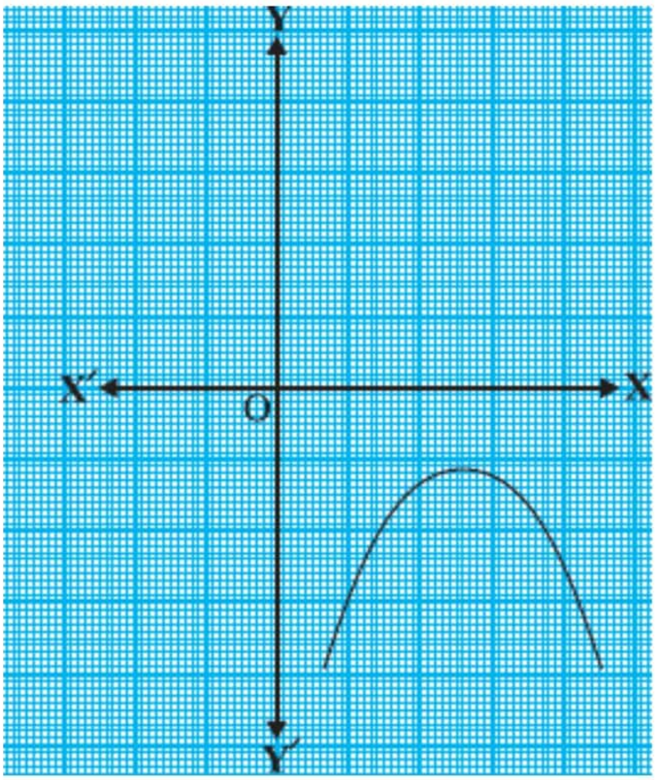
Answer: no zero

How many zeroes do the following polynomials have ?

(d)

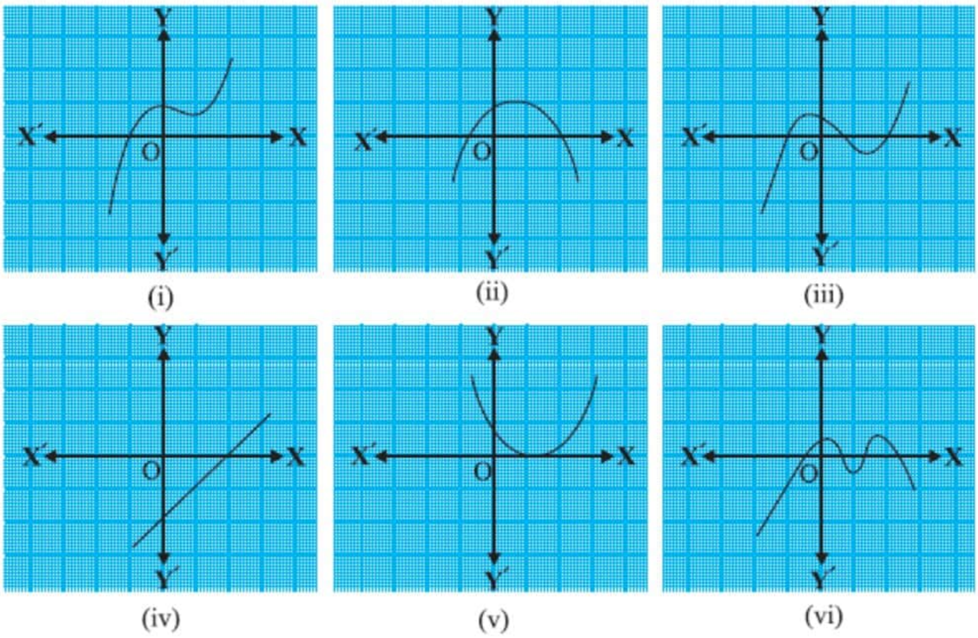


(d)



Answer: no zero

Some more............

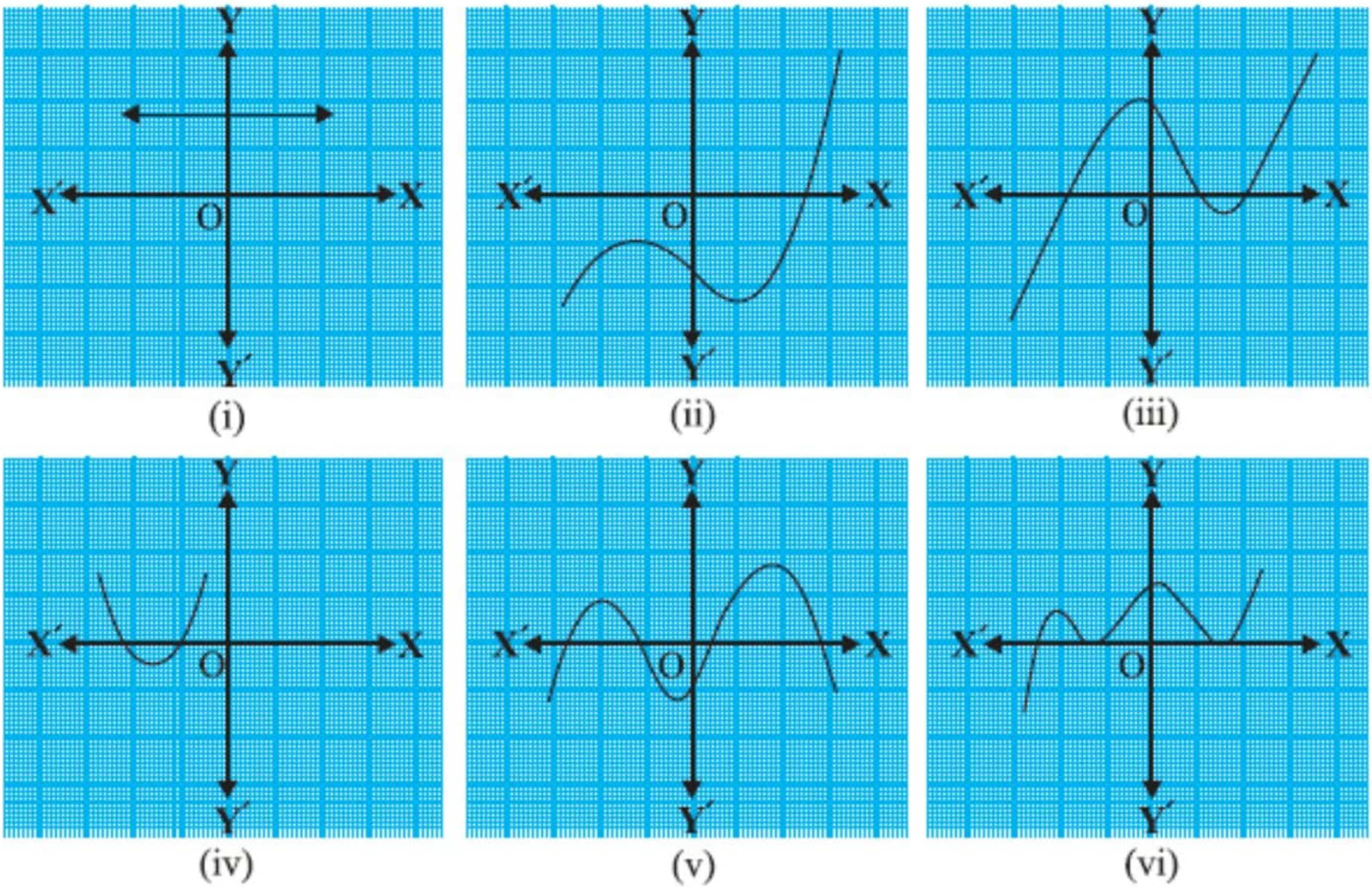


TESTING TIME

Solve Ex 2.1 and see if you can get the answers.

EXERCISE 2.1

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



WHAT'S NEW THIS TIME?

- Geometrical meaning of the zeroes of a polynomial
- Relationship between the zeroes and coefficients of a polynomial
- Division Algorithm for Polynomials

Objectives

- To find sum/product of roots *without* knowing the actual value of x.

- Use the sum/product of roots to solve for other results

Derivation of Formula

Given α and β are roots of a quadratic equation, we can infer that

$x_1 = \alpha$ and $x_2 = \beta$, where α and β are just some numbers.

For example, $x^2 - 5x + 6 = 0$ has roots α and β → means that $\alpha = 2$ and $\beta = 3$

However...

Sometimes, we do not need the values of x to help us solve the problem.

Knowing the relationship between the quadratic equation and its roots helps us save time

Derivation of Formula

$ax^2 + bx + c = 0$ has roots α and β -----------(1)

This tells us that: $(x - \alpha)(x - \beta) = 0$

Expanding, we have:

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

So, if we compare the above with (1):

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Result: $\alpha\beta = \frac{c}{a}$, $\alpha + \beta = -\frac{b}{a}$

Scanned with CamScanner

In other words...

$ax^2 + bx + c = 0$ has roots α and β

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

Let us try

Q1. Find a quadratic polynomial , the sum and product of whose zeroes are 3 and 2 respectively.

Q2. Find the zeroes of the polynomial $x^2 - 3x + 4$ and verify the relationship between the zeroes and the coefficients..

Q3. Find a quadratic polynomial , whose zeroes are 1,1 respectively.

Application – Example 1

$2x^2 + 6x - 3 = 0$ has roots α and β, find the value of

(a) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

Without finding α and β, we know the value for α + β and αβ

α + β = – 3 and αβ = – 3/2

Hence, $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta + \alpha}{\alpha\beta} = \dfrac{-3}{-\dfrac{3}{2}} = 2$

Application – Example 1—contd.

$2x^2 + 6x - 3 = 0$ has roots α and β, find the value of

(b)

We already know that: $\alpha + \beta = -3$ and $\alpha\beta = -3/2$

Expanding (b), we have

$$(2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2\alpha + 2\beta + 1$$
$$= 4\alpha\beta + 2(\alpha + \beta) + 1$$
$$= 4(-\frac{3}{2}) + 2(-3) + 1$$
$$= -11$$

Application – Example 2

$x^2 - 2x - 4 = 0$ has roots α and β, find the value of

Step 1: $\alpha + \beta = 2$ and $\alpha\beta = -4$

To find this, we use the algebraic property

$$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$

Rearranging:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
$$= (2)^2 - 2(-4)$$
$$= 12$$

Independent Practice

$2x^2 + 4x - 1 = 0$ has roots α and β, form the equation with roots α^2 and β^2

Remember!

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$

So in order to get the equation, find $\alpha^2 + \beta^2$ and $\alpha^2\beta^2$

DIVISION ALGORITHM

Ways to divide a polynomial

Factor Theorem

- Use zero of a polynomial to find the other factor
- Factorize it to get the zeroes

Long Division

- Actual division of two polynomials
- Factorize the quotient to get the remaining zeroes

DIVISION ALGORITHM

Dividend = (Quotient × Divisor) + Remainder

Example 1:

Divide $2x^2 + 3x + 1$ by $x + 2$.

$$
\begin{array}{r|l}
 & \quad 2x - 1 \\
\hline
x + 2 & 2x^2 + 3x + 1 \\
 & \underline{\overset{-}{2x^2} \overset{-}{+} 4x} \\
 & \qquad -x + 1 \\
 & \qquad \underline{\overset{+}{-}x \overset{+}{-} 2} \\
 & \qquad\qquad \underline{3}
\end{array}
$$

Example 2:

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

$$\begin{array}{r|l} & \quad\; x - 2 \\ -x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\ & -x^3 + x^2 - x \\ & + \quad - \quad + \\ \hline & \quad 2x^2 - 2x + 5 \\ & \quad 2x^2 - 2x + 2 \\ & \quad\;\; - \quad + \quad - \\ \hline & \qquad\quad 3 \end{array}$$

Divisor × Quotient + Remainder

$= (-x^2 + x - 1)(x - 2) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

= Dividend

Example: 3

Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

$$
\begin{array}{r l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 2 \,\big) & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & 2x^4 \qquad\quad - 4x^2 \\
 & - \qquad\qquad\quad + \\
\hline
 & -3x^3 + x^2 + 6x - 2 \\
 & -3x^3 \qquad + 6x \\
 & + \qquad\qquad - \\
\hline
 & x^2 \qquad - 2 \\
 & x^2 \qquad - 2 \\
 & - \qquad\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Example 3…contd..

So, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$.

Now, by splitting $-3x$, we factorise $2x^2 - 3x + 1$ as $(2x - 1)(x - 1)$. So, its zeroes are given by $x = \frac{1}{2}$ and $x = 1$. Therefore, the zeroes of the given polynomial are

$\sqrt{2},\ -\sqrt{2},\ \frac{1}{2},$ and 1.

Let us try…..

- Obtain all other zeroes of the polynomial $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$

What have we learnt so far.....

Zeroes of a polynomial

- Geometrical representation

Relationship between zeroes and coefficients

- Sum and product of zeroes
- Forming a polynomial

Division Algorithm

- Long Division
- Finding all zeroes of the given polynomial

Just a closing thought....

"Do not worry about your difficulties in Mathematics. I assure you, mine are still greater..."

---- Albert Einstein