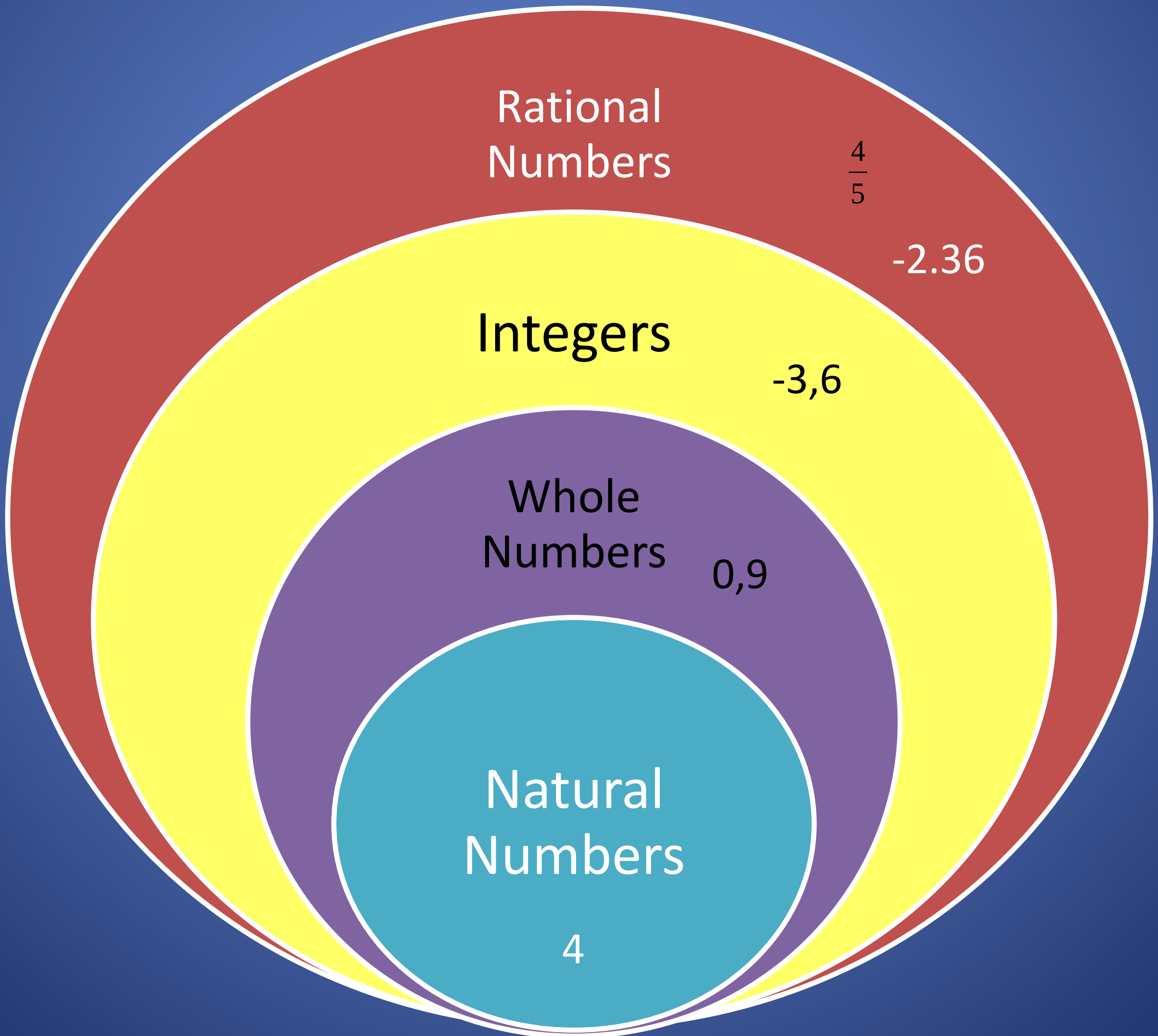
NUMBER SYSTEM

BRIEF REVIEW : RATIONAL NUMBERS



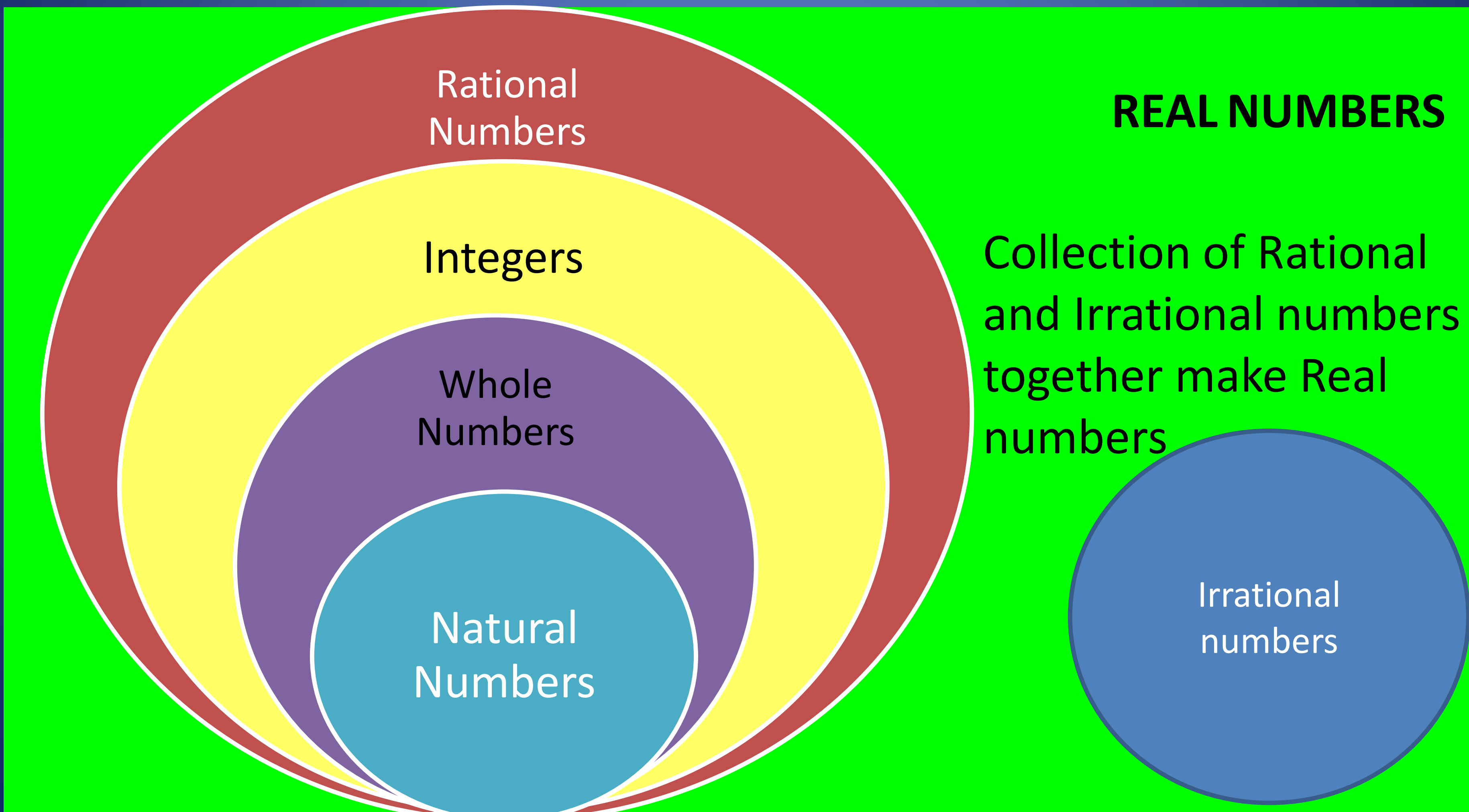
Objective: To study

- Introduction of Irrational numbers
- Real Numbers as Rationals and Irrationals
- Decimal expansion of Real Numbers
- Representation Real Numbers on the number line
- Operations on Real Numbers
- Rationalisation of Real Numbbers
- Laws of Exponents for Real Numbers

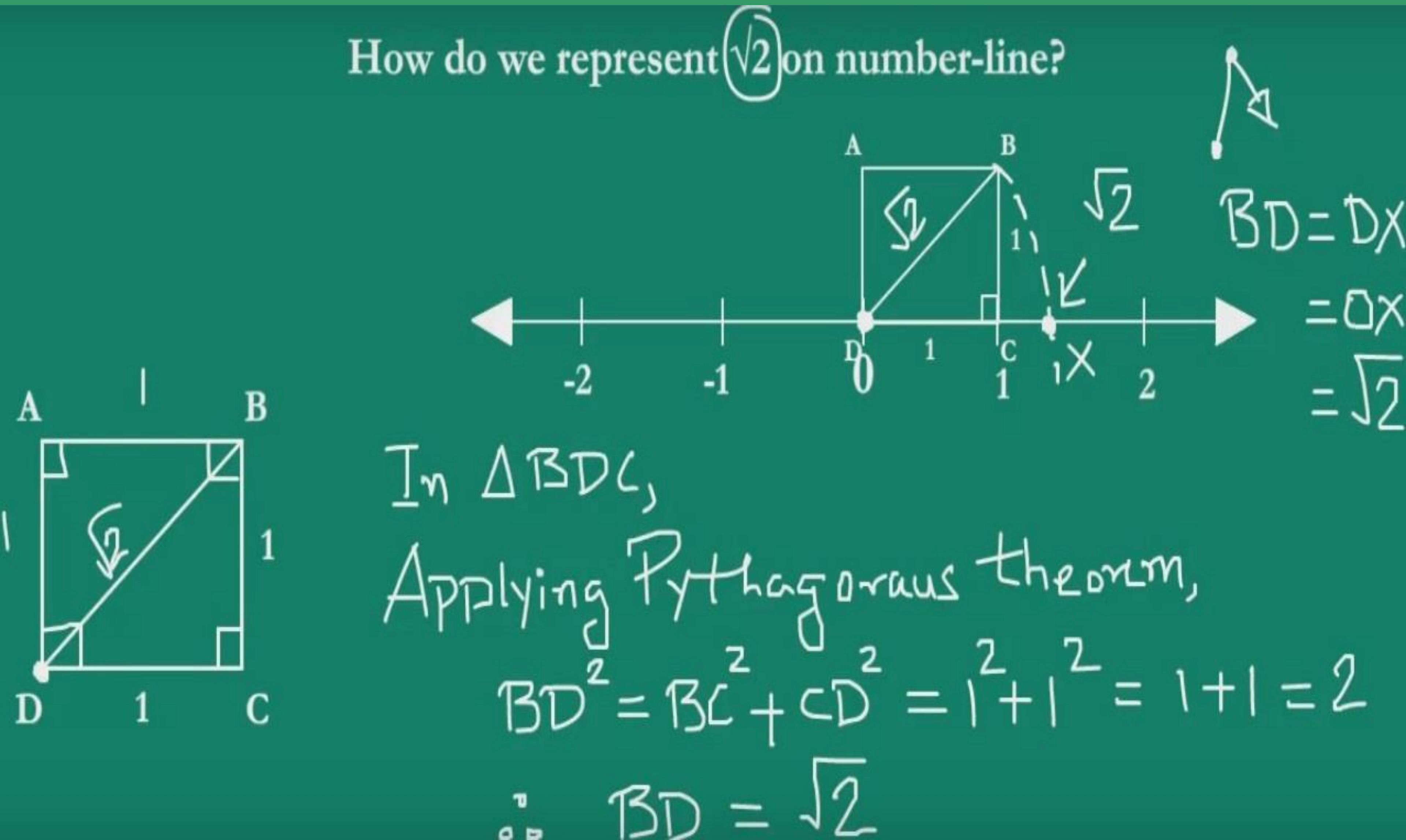
IRRATIONAL NUMBERS

- A number ‘s’ is called irrational, if it can not be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- Example – square root of non- perfect square numbers, Cube root of a number which is not a perfect cube, π
- A rational number can not be irrational and an irrational can not be rational. This means there is no number which is both rational as well as irrational.

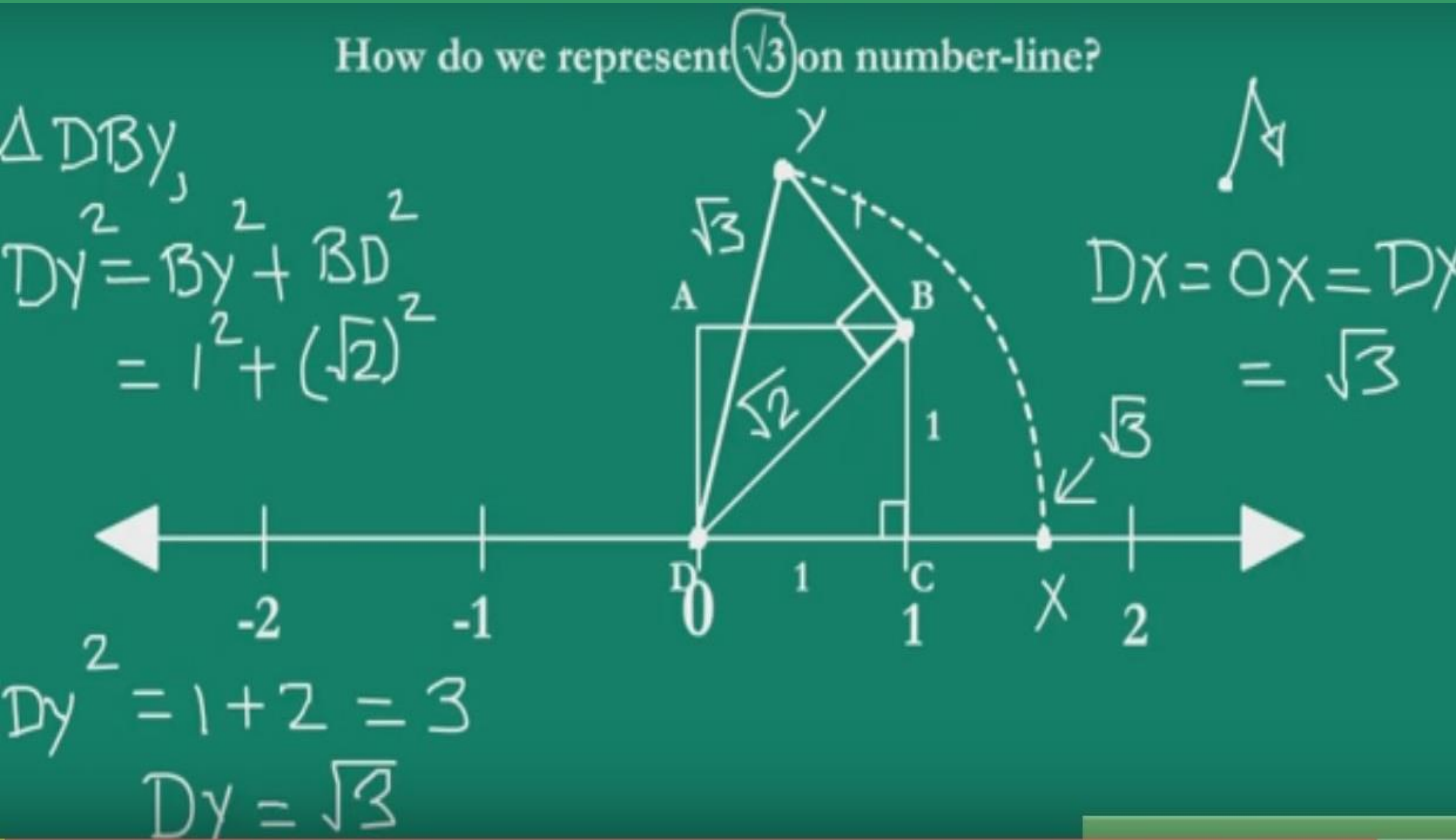
REAL NUMBERS



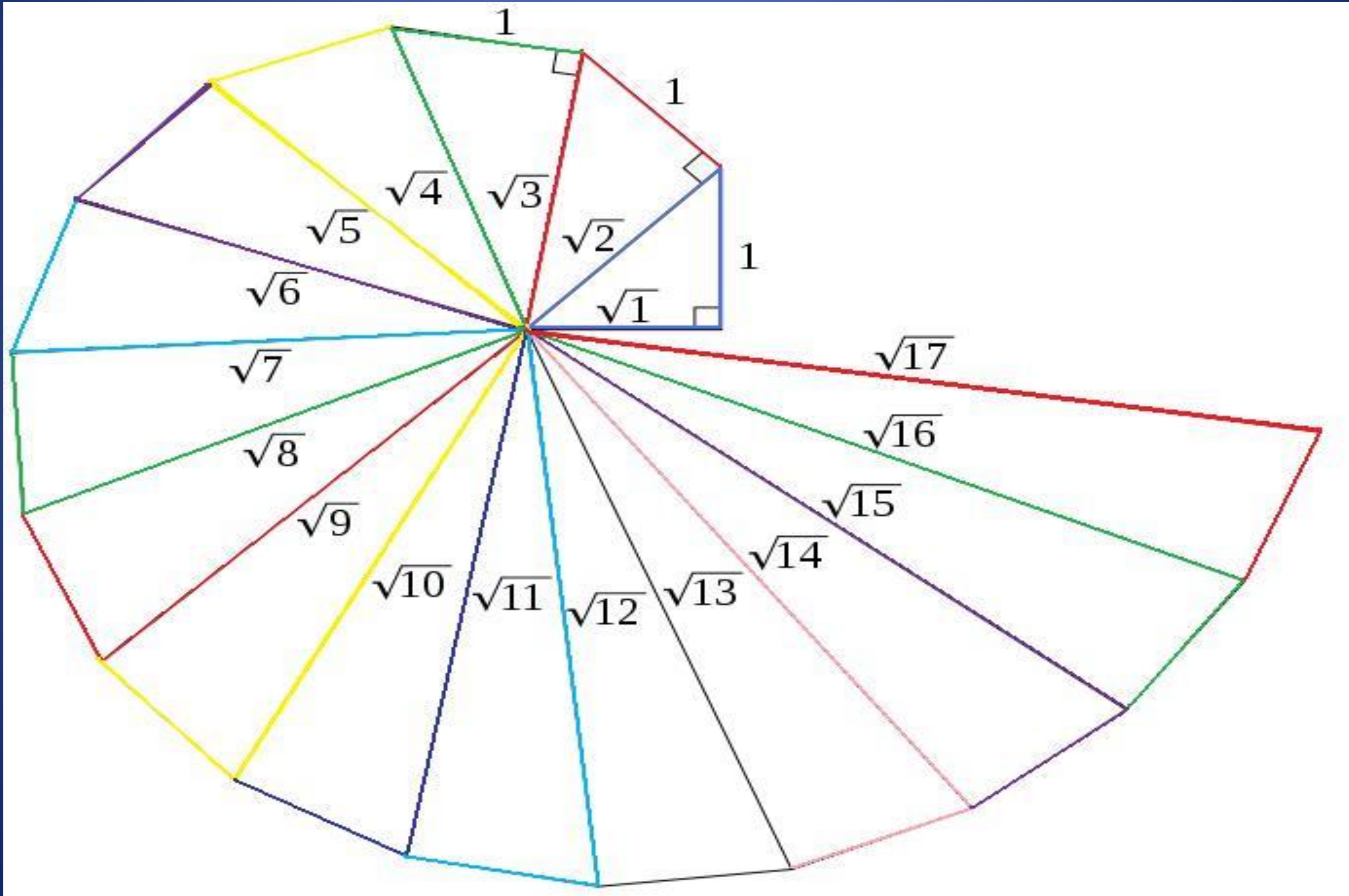
√2 ON NUMBER LINE



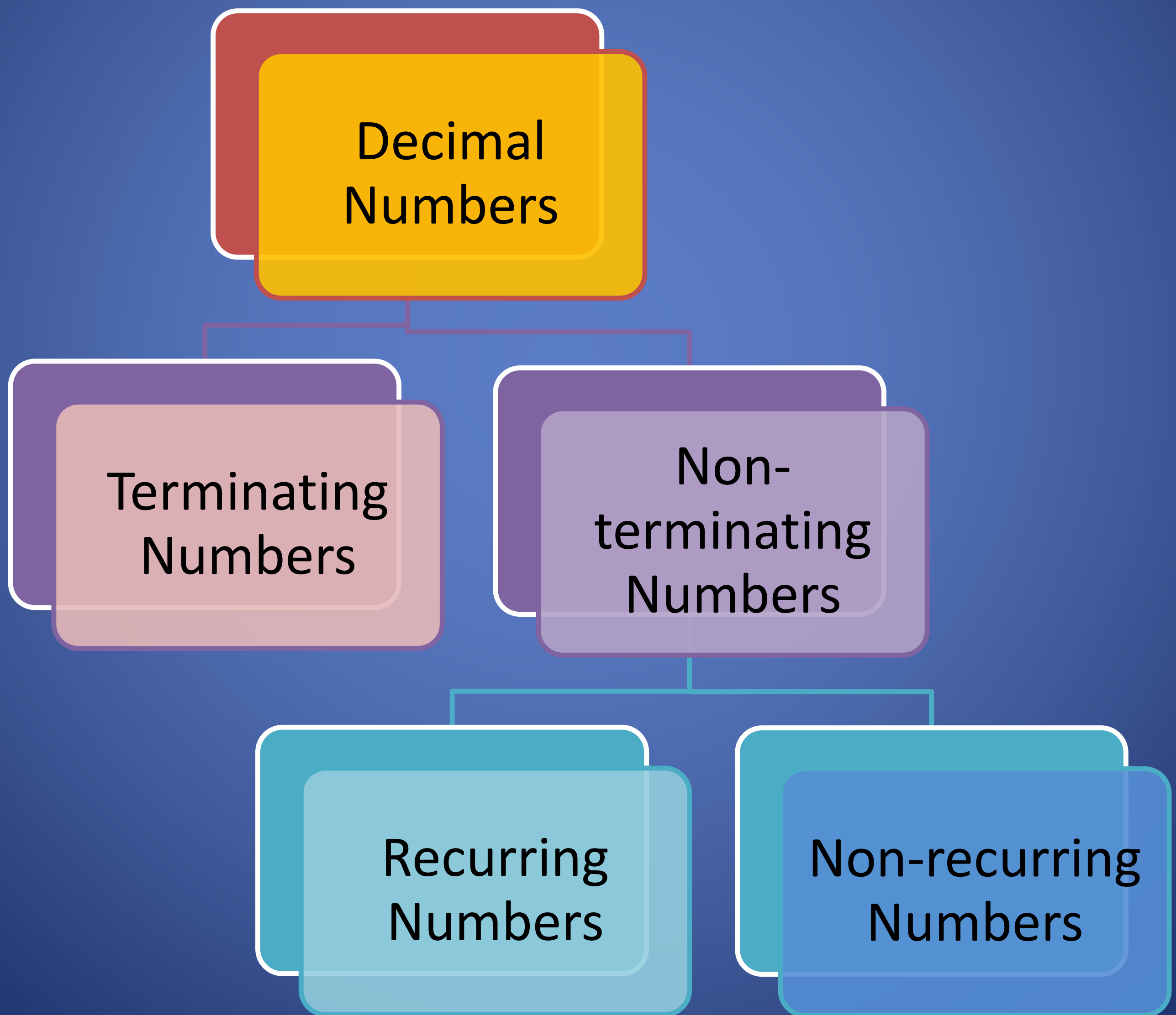
√3 ON NUMBER LINE



SQUARE ROOT SPIRAL



REAL NUMBERS AND THEIR DECIMAL EXPANSION



- All terminating and non terminating recurring decimals can be expressed in the form $\frac{p}{q}$ so they are RATIONAL NUMBERS.
- No non terminating non recurring number can be expressed as $\frac{p}{q}$ so they are IRRATIONAL NUMBERS

Q Show that 0.222.... Is rational.

Let $x = 0.222\ldots$

$10x = 2.22\ldots$

$9x = 2$

$x = 2/9$

Q Show that 0.375 is rational

$0.375 = 375 / 1000$

or $77 / 200$

Rational and Irrational Numbers

Rational Numbers

$$\frac{a}{b}$$

All terminating and repeating decimals can be expressed in this way so they are rational numbers.

Examples

$$\frac{4}{5} \qquad 2\frac{2}{3} = \frac{8}{3} \qquad 6 = \frac{6}{1} \qquad -3 = -\frac{3}{1} \qquad 2.7 = \frac{27}{10}$$

$$0.7 = \frac{7}{10} \qquad 0.625 = \frac{5}{8} \qquad 34.56 = \frac{3456}{100}$$

$$0.\dot{3} = \frac{1}{3} \qquad 0.\dot{2}\dot{7} = \frac{3}{11} \qquad 0.\dot{1}4285\dot{7} = \frac{1}{7}$$

Rational and Irrational Numbers

Determine whether the following are rational or irrational.

(a) 0.73 — rational

(b) $\sqrt{2}$ — irrational

(c) 0.666…. — rational

(d) 3.142 — rational

(e) $\sqrt{12.25}$ — irrational

(f) $\sqrt{7}$ — irrational

(g) $4+\sqrt{5}$ — irrational

(h) $(\sqrt[3]{2})^3+1$ — rational

(i) $16^{\frac{1}{2}}$ — rational

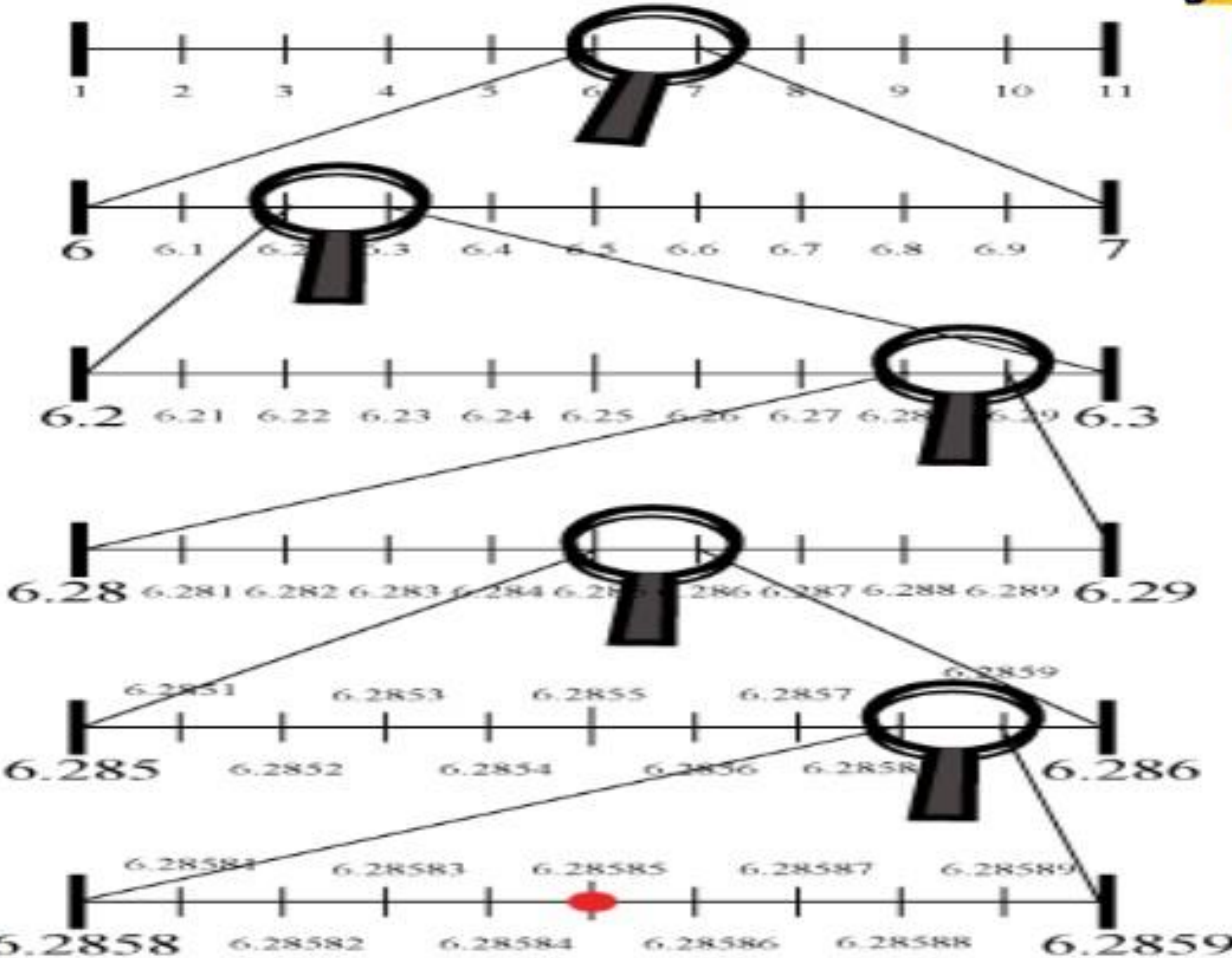
(j) $(\sqrt[3]{2})^2$ — irrational

(j) $(\sqrt{3}+1)(\sqrt{3+1})$ — irrational

(k) $(\sqrt{6}+1)(\sqrt{6}-1)$ — rational

(l) $(1+\sqrt{2})(1-\sqrt{2})$ — rational

- Representation Of Rational Numbers On A Number Line Using Successive Magnification



1 2 3 4 5 6 7 8 9 10 11
6 6.1 6.2 6.3 6.4 6.5 6.6 6.7 6.8 6.9 7
6.2 6.21 6.22 6.23 6.24 6.25 6.26 6.27 6.28 6.29 6.3
6.28 6.281 6.282 6.283 6.284 6.285 6.286 6.287 6.288 6.289 6.29
6.2851 6.2853 6.2855 6.2857 6.2859
6.285 6.2852 6.2854 6.2856 6.2858 6.286
6.28581 6.28583 6.28585 6.28587 6.28589
6.2858 6.28582 6.28584 6.28586 6.28588 6.2859

OPERATIONS ON REAL NUMBERS

- The sum or difference of a rational and an irrational number is irrational.
- The product or quotient of a non zero rational number with an irrational number is always irrational.
- Sum, difference, product and quotient of two irrational numbers may be rational or irrational

Rational and Irrational Numbers

Questions

State whether each of the following are rational or irrational.

a $\sqrt{6} \times \sqrt{7}$

irrational

b $\sqrt{20} \times \sqrt{5}$

rational

c $\sqrt{27} \times \sqrt{3}$

rational

d $\sqrt{4} \times \sqrt{3}$

irrational

e $\frac{\sqrt{32}}{\sqrt{8}}$

rational

f $\frac{\sqrt{44}}{\sqrt{11}}$

rational

g $\frac{\sqrt{18}}{\sqrt{2}}$

rational

h $\frac{\sqrt{25}}{\sqrt{5}}$

irrational

Rational and Irrational Numbers

Combining Rationals and Irrationals

Addition and subtraction of an integer to an irrational number gives another irrational number, as does multiplication and division.

$3\sqrt{8} + 2\sqrt{11}$

$\dfrac{\sqrt{3}}{5}$

$5\pi - 4$

$(\sqrt{3} + 5)(\sqrt{3} + 5)$

$3 + 10\sqrt{3} + 25$

$= 28 + 10\sqrt{3}$

$(\sqrt{6} + 2)(\sqrt{6} + 7)$

$6 + 9\sqrt{6} + 14$

$20 + 9\sqrt{6}$

REPRESENTATION OF √x ON NUMBER LINE

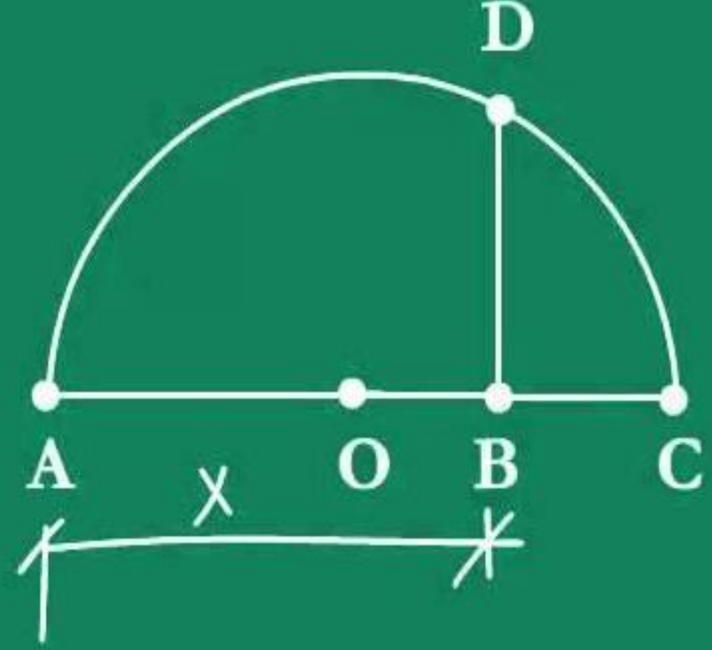
How to find out √x geometrically where x is any real number?

1. Take any point A. Then draw a line AB = x units.

2. Now extend AB to point C such that BC = 1 unit.

3. Now find out the mid-point of AC say point O. Using O as center draw a semi-circle with radius OC.

4. Draw a straight line from point B that is perpendicular to AC and intersecting semi-circle at point D.



Length BD = √x units.

RATIONALISING THE DENOMINATOR WITH SURD

Example 1 – Single Surd   Rationalise the denominator of: $\frac{2}{\sqrt{3}}$

Okay, so we don't like the look of that $\sqrt{3}$ on the bottom

What could we multiply it by to make it disappear?... Well, using Rule 2... how about by itself!

Be careful: Remember, whatever we multiply the bottom of the fraction by, we must also do to the top, otherwise the value of the fraction changes, so we will have changed the question!

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

Using our Rules of Fractions, we just multiply the tops together, and then the bottoms together →

$$2 \times \sqrt{3} = 2\sqrt{3}$$

And using Rule 2...

$$\sqrt{3} \times \sqrt{3} = 3$$

So, we are left with our answer! $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

And if you check them on the calculator, you will see they give the same answer!

ANOTHER EXAMPLE OF SURD

Example 2 – Surd with Other Numbers Rationalise the denominator of: $\frac{5}{3-\sqrt{2}}$

Okay, so let's multiply the top and the bottom of the fraction by... change the sign... $3+\sqrt{2}$

$$\frac{5}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

Again, we multiply tops and bottoms together, but we also use our methods of expanding brackets (see the Algebra section)

Tops → $5\times(3+\sqrt{2})$ → $15+5\sqrt{2}$

Bottoms Use FOIL → $(3-\sqrt{2})\times(3+\sqrt{2})$ → $9+3\sqrt{2}-3\sqrt{2}-2$

Now look what happens when we collect up our terms and simplify → $9-2=7$

The middle two terms cancel out, and we are left with a very nice (and rationalised) denominator!

So... our answer must be... $\frac{5}{3-\sqrt{2}} = \frac{15+5\sqrt{2}}{7}$

And if you check them on the calculator, you will see they give the same answer!

LAWS OF EXPONENTS FOR REAL NUMBERS

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2x^3 = x^{2+3} = x^5$
$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times3} = x^6$
$(xy)^n = x^ny^n$	$(xy)^3 = x^3y^3$
$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$

And the law about Fractional Exponents:

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $= (\sqrt[n]{x})^m$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$ $= (\sqrt[3]{x})^2$

SUMMARY

This chapter is based on real numbers, their decimal representation and their representation on number line.

Real numbers are the collection of all the rational and irrational numbers. It is denoted by the symbol ‘R’.

Every real number can be represented on the number line with a unique point on the number line. Also, every point on the number line represents a unique real number. That is why we call the number line a real number line.

Real Number System Tree Diagram